Instructions. This is a closed-book, closed-notes, closed-other-people, no-calculators exam. There are 9 problems, worth 10 points each. Make sure you justify your solutions for questions 2, 5 and 6; answers alone will receive little or no credit.

1. Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4\}$, and $A_3 = \{1, 3, 5\}$. Write down the following sets.

   (a) $A_1 \times A_2$

   (b) $\mathcal{P}(A_3)$

   (c) $A_1 - (A_3 - A_2)$
2. Identify each statement as true or false. Briefly justify your answers.

(a) \( \emptyset = \{ \} \)

(b) \( \emptyset \in \{ \} \)

(c) \( \emptyset \subseteq \{ \} \)
3. (a) Show using truth tables that \( \sim (P \Rightarrow Q) = P \land \sim Q \).

(b) Give an example (in ordinary English) of a true statement whose converse is false. Be sure to identify which is the original and which is the converse. Explain why the first is true and the second is false. Be original.
4. Translate the following sentences into symbolic logic. Do not use any words. Use the symbol \( \mathbb{P} \) for the set of prime numbers.

(a) If \( p \) is a prime number then \( \sqrt{p} \) is not a rational number.

(b) There exists a prime number \( q \) such that \( q/2 \) is an integer.
5. Translate the following into English sentences. Say whether each is true or false. Justify your answers.

(a) \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y > x \).

(b) \( \exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y > x \).
6. Consider $c$ to be an integer in the open sentence:

$$\exists t \in \mathbb{N}, c \mid t^2 \text{ and } c \nmid t.$$ 

(a) Negate the open sentence.

(b) Is there a value of $c$ that makes the original sentence true? Why or why not?

(c) Is there a value of $c$ that makes the original sentence false? Why or why not?
7. Use a direct proof to prove the following:

Proposition: Suppose \( a, b \in \mathbb{Z} \). If \( a \mid b \) then \( a^2 \mid b^2 \).
8. Use a contrapositive proof to prove:

   Proposition: Suppose $a \in \mathbb{Z}$. If $a^2$ is not divisible by 4 then $a$ is odd.
9. Prove using either direct or contrapositive proof:
   If $n$ is odd then $8 \mid (n^2 + 1)$. 