Algebra II

Homework 7

Due Friday, April 9

Section 2.6: 26, 27

B. If R is a commutative ring with unity, two elements a and b in R are **associates** if there exists $u \in R^*$ such that

$$a = ub.$$

1. Prove that the relation $\sim$ on R defined by $a \sim b$ if and only if a and b are associates is an equivalence relation.
2. Suppore F is a field. Prove that if $a(x), b(x) \in F[x]$, then $a(x)$ and $b(x)$ are associates if and only if $a(x)$ divides $b(x)$ and $\deg(a(x)) = \deg(b(x))$.
3. Recall that a polynomial is **monic** if its leading coefficient is 1. Prove that each equivalence class of $\sim$ in $F[x]$ except the class of 0 contains exactly one monic element.