

Algebra II
Homework 5
Due Monday, March 29

Section 22: 27

Section 23: 10, 14, 31, 34

- A. List all the irreducible polynomials of degree less than or equal to 3 in $\mathbf{Z}_2[x]$.

Hint. Remember that if a polynomial of degree 2 or 3 factors into polynomials of smaller degree, then one of those factors must have degree 1.

Note that constant polynomials are not irreducible.

- B. List all the irreducible polynomials of degree 4 and 5 in $\mathbf{Z}_2[x]$

Hint. When you extend your list to include polynomials of degree 4 and 5, it is no longer sufficient to check for roots, since there are reducible polynomials with no roots. To resolve this problem, take a page from Eratosthenes's book; if a polynomial of degree 4 or 5 has no degree 1 irreducible factors, what irreducible factors could it have?

Recall that the sieve of Eratosthenes finds the prime (i.e., irreducible) numbers less than n by listing the numbers up to n and inductively crossing off the multiples of each prime in turn until only primes remain.