Algebra 2

Homework 2

Due Friday, February 5

Section 18: 40, 41, 55, 56a

A. Let $S = \{m/2^k : m \in \mathbf{Z}, k \in \mathbf{Z}_{\geq 0}\}$, which is one of the subrings of $\mathbf{R}$ we considered in class. Prove that S is the smallest subring of $\mathbf{R}$ that contains $\frac{1}{2}$. In other words, prove that if T is a subring of $\mathbf{R}$ and $\frac{1}{2} \in T$, then $S \subseteq T$.

B. Suppose that R is a ring with unity and $\phi\colon R \longrightarrow S$ is a ring homomorphism. Prove that $\phi[R]$ is a ring with unity and that $\phi(1_R)$ is the multiplicative identity of $\phi[R]$.

   Remark. As in our example in class of $\tau\colon \mathbf{Z} \longrightarrow \mathbf{Z} \times \mathbf{Z}$ defined by $\tau(a) = (a, 0)$, $\phi(1_R)$ might not be the multiplicative identity of S.