

Math 322
Explorations of Rings and Fields

1. Suppose that R is a commutative ring with unity and that the characteristic of R is a composite number. Prove that R cannot be an integral domain.

Remark. If the general case is too abstract to start with, try thinking about the case where R is an arbitrary ring of characteristic 6.

2. We will now investigate the ring $E = \{\bar{0}, \bar{1}, \alpha, \beta\}$ where $\bar{0}, \bar{1} \in \mathbf{Z}_2$, α is a root of the polynomial $x^2 + x + \bar{1}$ with coefficients in \mathbf{Z}_2 , and $\beta = \alpha + \bar{1}$. You may assume the associative, commutative, and distributive properties, and that $\bar{0}$ and $\bar{1}$ are the additive and multiplicative identities. Note that this implies that E has characteristic 2.

- (a) Make an addition table for E . (This proves that E is closed under addition.)
- (b) Make a multiplication table for E . (This proves that E is closed under multiplication.)
- (c) Show that β is also a root of $x^2 + x + \bar{1}$.
- (d) Prove that E is a field.
- (e) Find a quadratic polynomial with coefficients in E that has no roots in E .