Math 151 Final Exam

1. (12 points)

   Find the following limits.

   (a) $\lim_{x\to 1} \dfrac{x-1}{x^2-7x+6}$

   (b) $\lim_{x\to 2} \dfrac{x-1}{x^2-5x+6}$

   (c) $\lim_{x\to -\infty} \dfrac{x^2-27x}{5-4x^2}$

2. (6 points)

   Using the definition of the derivative, find the derivative of $g(x) = \sqrt{2x+1}$.

3. (20 points)

   Find the following derivatives.

   (a) $f(x) = x^3 + x^2 - 5\sqrt{x}$; find $f''(x)$.

   (b) $g(t) = \sec(\pi - t^2)$; find $g'(t)$.

   (c) $y = \dfrac{w+9}{w^2-3}$; find $\frac{dy}{dw}$.

   (d) $G(x) = \int_3^x \tan\sqrt{t}\, dt$; find $G'(x)$.

4. (6 points)

   Find the tangent line to the curve $x^3 - 6xy + y^2 = 2x - 7$ at the point $(2, 1)$.

5. (8 points)

   Find the maximum and minimum values of $f(x) = x^4 - 2x^3$ on the interval $[-1, 4]$.

6. (16 points)

   Compute the following.

   (a) $\int_0^3 (x^2 - 1);dx$

   (b) $\int \sec^2(5t)\, dt$

   (c) The Riemann sum for $f(x) = x - x^2$ on the interval $[-2, 4]$ with three pieces and right endpoints.

   (d) The area between $y = 6x - x^2$ and the x-axis.

   Hint. To figure out where to start and end your integral, you might find drawing a graph helpful.

7. (12 points)

   Susan plans to build a box with a square bottom, an open top, and a capacity of 1000 in^3. The material for the sides costs \$1 per square inch, but the bottom will be made of sturdier material that costs \$2 per square inch.

   What dimensions should Susan give the box so that she can build it as cheaply as possible?

8. (12 points)

   A clown is shot out of a cannon. Her vertical position in feet as a function of time in seconds is

   $$c(t) = 96t - 16t^2.$$

   (a) What is the clown's upward velocity as a function of time?

   (b) When is the clown's upward velocity 0 ft/sec?

   (c) How high does the clown fly?

   (d) The clown is aimed to fall into a net 80 feet above the ground. How fast is the clown falling when she is about to hit the net?

9. (8 points)

   (a) State the Mean Value Theorem.

   (b) For $f(x) = \frac{1}{x}$ on the interval $[1, 9]$, give the number c guaranteed by the Mean Value Theorem.