Math 151
Exploring Applications of Derivatives

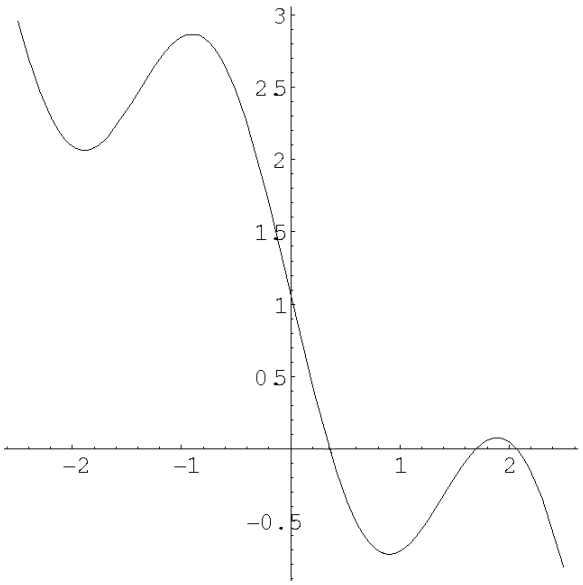
1. Verify that the function $f(x) = \sqrt{x-3}$ on the interval $[4, 28]$ satisfies the conditions for the Mean Value Theorem, and find the number c in $[4, 28]$ guaranteed by the Theorem.

2. Caitlin plans to build a box with a square bottom and an open top. The material for the sides costs 2¢ per square inch, but the bottom will be made of sturdier material that costs 3¢ per square inch. She has $9.00 budgeted for the box materials.

   What is the largest volume Caitlin can give her box?

   Hint. You will probably be happier if you measure the cost in cents, as opposed to dollars.

3. A local television station is running a *Lost* marathon, and Chris's Candy Store is buying ad time during the marathon. Market research shows that every time Chris airs his ad, he gains 1,000 new customers. However, once the ad airs t times, a total of $20t^2$ of these potential customers have become so sick of the ad that they vow never to shop at Chris's. How many ads should Chris run to get the most new customers?



4. Above is the graph of $f'(x)$. At what values of x does $f(x)$ have a local maximum? A local minimum? An inflection point?

   Note that the graph here is NOT the graph of $f(x)$. So, for example, it is not true that $f(x)$ has a local maximum at $x = -0.8$.

5. Let $g(x) = 2 - \frac{6}{x} + \frac{6}{x^2} = \frac{2x^2-6x+6}{x^2}$.

   (a) Find all the asymptotes of the graph $y = g(x)$.

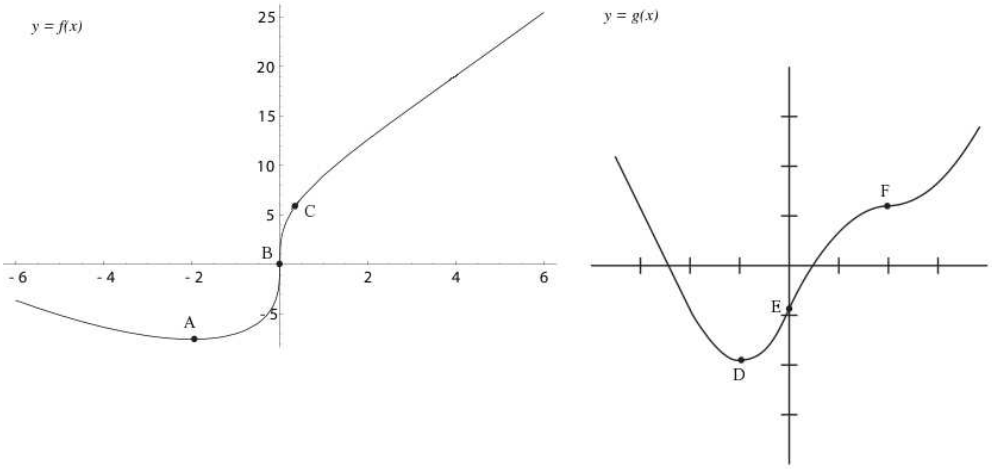
   (b) Find the intervals on which g is increasing and on which g is decreasing.

   (c) Find the intervals on which g is concave up and on which g is concave down.

   (d) Sketch the graph $y = g(x)$. Be sure to label all asymptotes, local maxima, local minima, and inflection points.

   Hint. For some parts, it is easier to add up separate fractions, as in the first formula; for others, it is easier to bring everything over a common denominator, as in the second formula.

6. Below are the graphs of two functions.



   At points A, B, and C, is $f'(x)$ positive, negative, zero, or undefined? Is $f''(x)$ positive, negative, zero, or undefined?

   At points D, E, and F, is $g'(x)$ positive, negative, zero, or undefined? Is $g''(x)$ positive, negative, zero, or undefined?