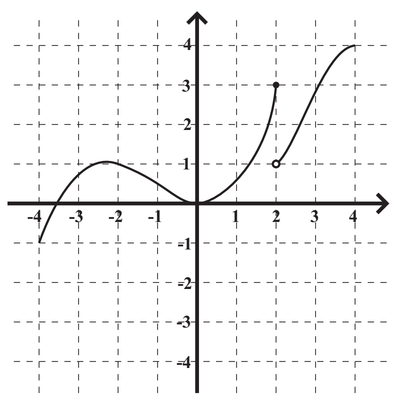
Math 151
Exploring Integrals
Numerical Solutions

1. Let $f(x) = x^2 - 3$. Compute the Riemann sum on the interval $[-2, 10]\ldots$

   (a) ...with 3 pieces and midpoints.
   $f(0) \cdot 4 + f(4) \cdot 4 + f(8) \cdot 4 = 284$

   (b) ...with 4 pieces and right endpoints.
   $f(1) \cdot 3 + f(4) \cdot 3 + f(7) \cdot 3 + f(10) \cdot 3 = 462$

2. For the function graphed below, give the Riemann sums using left endpoints and using right endpoints with four equal subintervals.



   Left Endpoint Sum $-1 \cdot 2 + 1 \cdot 2 + 0 \cdot 2 + 3 \cdot 2 = 6$

   Right Endpoint Sum $1 \cdot 2 + 0 \cdot 2 + 3 \cdot 2 + 4 \cdot 2 = 16$

3. Suppose a car is decelerating from a velocity of 120 ft/sec at 20 ft/sec^2. How far does the car travel before coming to a full stop?

   $a(t) = -20$

   $v(0) = 120$, and $v(t) = -20t + 120$

   The car stops when $v(t) = 0$ at $t = 6$.

   The car travels $\int_0^6 (-20t + 120)\, dt = \left[-10t^2 + 120t\right]_0^6 = 360$ ft.

4. Compute the following.

   (a) $\int (x^4 - 2x^2 + 1)\, dx = \frac{x^5}{5} - \frac{x^3}{3} + x + C$

   (b) $\int \cos 3\theta\, d\theta = \frac{1}{3} \sin 3\theta + C$

(c) $\int_{-1}^{8} \sqrt[3]{t}\, dt = \frac{45}{4}$

(d) $\int_{-3}^{3} x^3\, dx = 0$

(e) $\frac{d}{dx}\int_{3}^{x} \sec t\, dt = \sec(x)$

(f) $\frac{d}{dx}\int_{3}^{x^2} \sec t\, dt = \sec(x^2) \cdot 2x$

Hint. What is $\frac{d}{dy}\int_{3}^{y} \sec t\, dt$?

$\frac{d}{dy}\int_{3}^{y} \sec t\, dt = \sec y$, so if $y = x^2$, the chain rule tells us that

$$\frac{d}{dx}\int_{3}^{x^2} \sec t\, dt = \frac{d}{dy}\int_{3}^{x^2} \sec t\, dt \cdot \frac{dy}{dx} = \sec(x^2) \cdot 2x$$