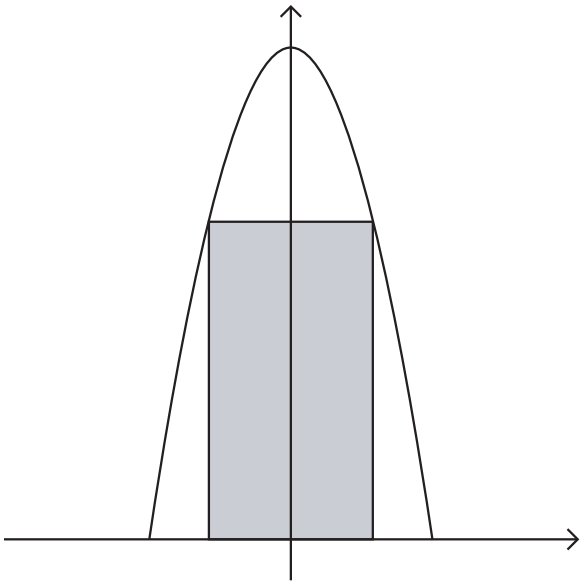
Math 151
Supplemental Exercises

1. A rectangle with its base along the x-axis has its top vertices on the parabola $4 - x^2$. What is the largest possible area of the rectangle?



If (x, y) is the upper right corner of the rectangle, the rectangle has base $2x$ (because it extends to the left and right of the y-axis) and the rectangle has height y.

Area of rectangle $= A = 2xy = 2x(4 - x^2) = 8x - 2x^3$.

The x value is in the interval $[0, 2]$, since it is between the y-axis and the positive x-intercept of the parabola.

$$A'(x) = 8 - 6x^2$$

The critical numbers are $x = \pm 2/\sqrt{3}$, but only $2/\sqrt{3}$ is in the interval $[0, 2]$.

x	$A(x)$
0	0
$2/\sqrt{3}$	$32/(3\sqrt{3})$
2	0

So the largest possible area is $32/(3\sqrt{3})$.

2. What is the general antiderivative of $3\sin x - \cos(5x)$?

$-3\cos x - (1/5)\sin(5x) + C$

3. Suppose $f'(x) = x^3 - 17$ and $f(4) = 10$. What is $f(x)$?

$f(x) = (1/4)x^4 - 17x + C$ for some value of C.

$10 = f(4) = (1/4)x^4 + 17 \cdot 4 + C = -4 + C$, which means $C = 14$.

Thus, $f(x) = (1/4)x^4 - 17x + 14$.

4. A car is decelerating at a constant rate of 20 ft/sec^2. If the car is going 90 ft/sec when the clock starts at $t = 0$, how far does the car move before it stops?

Acceleration $= a(t) = -20$

Velocity $= v(t) = -20t + C$, and $v(0) = 90$, so $v(t) = -20t + 90$.

The car stops when $v(t) = -20t + 90 = 0$, which is at $t = 4.5$ seconds.

Position $= s(t) = -10t^2 + 90t + D$, so the total distance the car travels is

$$s(4.5) - s(0) = 202.5 + D - D = 205.5 \text{ ft.}$$