Math 151

In-class Worksheet

1. Find the following derivatives.

   (a) $f(x) = \frac{x^3+4x^2-x-17}{x^2+1}$; $f'(x) = \frac{(x^2+1)(3x^2+8x-1)-(x^3+4x^2-x-17)(2x)}{(x^2+1)^2}$

   (b) $y = \sin(x^3) + \cos(\pi^3)$; $\frac{dy}{dx} = \cos(x^3) + 0$

   (c) $y = x^6 - 4x^5 + x^2 - 17x + 106$; $\frac{d^2y}{dx^2} = 30x^4 - 80x^3 + 2$

   (d) $s = \sin\sqrt{t^2+4}$; $\frac{ds}{dt} = \cos\sqrt{t^2+4} \cdot \frac{1}{2}(t^2+4)^{-1/2} \cdot 2t$

   (e) $g(x) = \tan x$; $g''(x) = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$

   (f) $h(t) = t^2\sqrt[3]{(t^3+17)^4}$; $h'(t) = t^2\frac{4}{3}(t^3+17)^{1/3} \cdot (3t^2) + (t^3+17)^{4/3} \cdot 2t$

2. Here is a table of values for a couple of functions and their derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	1	4	5	-4
0	2	-2	3	7
2	4	8	1	9
3	2	-7	-4	4
4	1	-1	-2	8

   For $h(x) = g(f(x))$, $j(x) = \frac{f(x)}{g(x)}$, and $k(x) = f(x^2)$, compute:

   (a) $h'(0) = g'(f(0))f'(0) = -18$

   (b) $j'(3) = \frac{5}{4}$

   (c) $k'(2) = f'(2^2)(2 \cdot 2) = -4$

3. Find the tangent line to the curve

$$x^3y^2 + x^2 - y = 3$$

   at the point $(1, 2)$.

   $y - 2 = \frac{-14}{3}(x - 1)$

4. Find the maximum and minimum values of the given functions.

   (a) $f(x) = x^3 - 12x + 7$ on the interval $[-3, 3]$
   critical numbers: ± 2
   maximum $= f(-2) = 23$
   minimum $= f(2) = -9$

   (b) $g(x) = \sqrt[3]{(x-4)^2}$ on the interval $[3, 12]$
   critical number: 4
   Note that $g'(4) \neq 0$, but 4 is a critical number because $g'(4)$ is undefined.
   maximum $= g(12) = 4$
   minimum $= g(4) = 0$

   (c) $h(x) = 1/x$ on the interval $[-1, 1]$
   no maximum or minimum value
   Note that because $h(x)$ has an asymptote at $x = 0$ and is thus not continuous, the closed interval method does not work.

5. The graphs in the righthand column represent the derivatives of the functions graphed in the lefthand column. Match each function with its derivative.

   $a' = z$

   $b' = x$

   $c' = w$

   $d' = v$

   $e' = y$