Math 151
Exploring Limits and Continuity
Numerical Solutions

1. Find the following limits.

(a) $\lim_{x\to -1} \dfrac{x^2+2x+1}{x^2-5} = 0$

(b) $\lim_{x\to 3} \dfrac{x^2-x-6}{x^2-5x+6} = 5$

(c) $\lim_{x\to 3^-} \dfrac{x+2}{x-3} = -\infty$

(d) $\lim_{x\to 3^-} \dfrac{x^2-9}{|x-1|-2} = 6$

(e) $\lim_{x\to 1^+} \dfrac{x^2-4x+3}{|1-x| = -2}$

(f) $\lim_{z\to 4} \dfrac{4-z}{2-\sqrt{z}} = 4$

2. Suppose a car is moving, and at time t measured in seconds, the car's position is $p(t) = 4t+t^2$ in feet.

   (a) What is the car's average velocity between $t = 3$ sec and $t = 4$ sec?
   11 ft/sec

   (b) What is the car's average velocity between $t = 2.9$ sec and $t = 3$ sec?
   9.9 ft/sec

   (c) **Without using formulas from physics or calculus**, how would you figure out the velocity of the car *at* $t = 3$ sec? Implement this method.

Velocity $= \lim_{t\to 3} \dfrac{4t+t^2-21}{t-3} = 10$ ft/sec

3. Above is the graph of the function $g(x)$. Find the following limits.

   (a) $\lim_{x\to -1} g(x) = -1$

   (b) $\lim_{x\to 3} g(x)$ does not exist.

   (c) $\lim_{x\to 3} (g(x))^2 = 4$

   (d) $\lim_{t\to 2} \dfrac{g(t)-\frac{3}{2}}{t-2} = g'(2) = \dfrac{1}{2}$

   (e) $\lim_{t\to -2} \dfrac{g(t)-1}{t+2} = g'(-2) = 0$

4. Let

$$f(x) = \begin{cases} x^2 & \text{when } x \geq 2 \\ ax-2 & \text{when } -1 \leq x < 2 \\ b-x^2 & \text{when } x < -1 \end{cases}$$

For what values of a and b is $f(x)$ continuous everywhere?

$a = 3$, $b = -4$.

5. (a) Does $x^3 - 3x + 1 = 7$ have a solution in the interval $(1, 3)$?
   Yes, by the Intermediate Value Theorem: $f(x) = x^3 - 3x + 1$ is continuous everywhere, and $f(1) < 7 < f(3)$.

   (b) Does $\frac{x+5}{x-2} = 0$ have a solution in the interval $(1, 3)$?
   No, because the only solution to this equation is $x = -5$.

   (c) Does either of the previous answers disagree with the Intermediate Value Theorem?
   No. The second function is discontinuous at $x = 2$.