Infinity Learner's Permit Exam

Carefully read and answer the following questions

1. Explain what is wrong with the following argument: Let $x = 1 + 2 + 4 + 8 + 16 + \cdots$. Then $2x = 2 + 4 + 8 + 16 + \cdots$. So

$$x = 2x - x = (2 + 4 + 8 + \cdots) - (1 + 2 + 4 + 8 + \cdots) = -1.$$

2. Explain why the set of natural numbers $\{1, 2, 3, 4, \ldots\}$ and the set of even numbers $\{2, 4, 6, 8, \ldots\}$ have the same cardinality.

3. Show with a list that there are infinitely many natural numbers that are not prime.

4. Show with a list that there are infinitely many real numbers that are not rational.

5. Name a set that has greater cardinality than the set of natural numbers.

6. Find the area of the region bounded by the curves $y = \frac{1}{x}$, $x = 1$, and the x- and y-axes.

7. Explain why $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \cdots = 1$.

∞. How many different sizes of infinity are there?