Infinity Learner’s Permit Exam

Carefully read and answer the following questions

1. Explain what is wrong with the following argument: Let \( x = 1 + 2 + 4 + 8 + 16 + \cdots \). Then \( 2x = 2 + 4 + 8 + 16 + \cdots \). So

\[
x = 2x - x = (2 + 4 + 8 + \cdots) - (1 + 2 + 4 + 8 + \cdots) = -1.
\]

2. Explain why the set of natural numbers \( \{1, 2, 3, 4, \ldots\} \) and the set of even numbers \( \{2, 4, 6, 8, \ldots\} \) have the same cardinality.

3. Show with a list that there are infinitely many natural numbers that are not prime.

4. Show with a list that there are infinitely many real numbers that are not rational.

5. Name a set that has greater cardinality than the set of natural numbers.

6. Find the area of the region bounded by the curves \( y = \frac{1}{x}, x = 1, \) and the \( x- \) and \( y- \) axes.

7. Explain why \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1. \)

\( \infty \). How many different sizes of infinity are there?