Constucting Discrete K-Surfaces

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(joint work with Tim Hoffmann, and Ulrich Pinkall)

Old and new approaches to the theorectical, computational and physical construction of discrete pseudospherical surfaces in $\mathbb{R}^3$ ("Discrete K-Surfaces") were presented.

1. Twisted Metal

G.T. Bennett introduced a "new mechanism" in 1903 [1]. The relation between this mechanism and discrete K-surfaces (including a mechanical description of the Bäckland transformation) is in [2]. The theory of discrete K-Surfaces was developed by Sauer [6], Wunderlich [8] and Bobenko-Pinkall [3]. In 1951 Wunderlich built a model which still exists today (FIGURE 1). It is also possible to build deformable prototypes by attaching ring terminals to the ends of butt splices for the edges; small bolts and wing nuts for the vertices. Besides assisting in research, these models are also useful in teaching. For example, the 1-dimensional curves on Bennett's isogram (the asymptotic coordinate lines) model discrete curves of constant torsion.

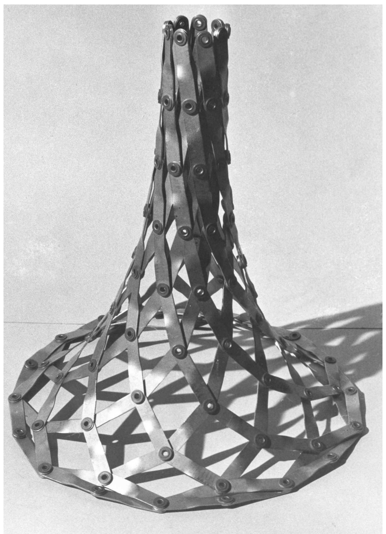

Figure 1. Wunderlich's Model (1951)

2. Sissors-Paper-Tape and Crocheting

On the other hand, in the early 1970's Thurston [7] outlined a completely different way of constructing certain discrete K-surfaces. These surfaces became well-known because of Henderson's paper models and Taimina's crocheted models [4]. A rather generic example is in FIGURE 2.

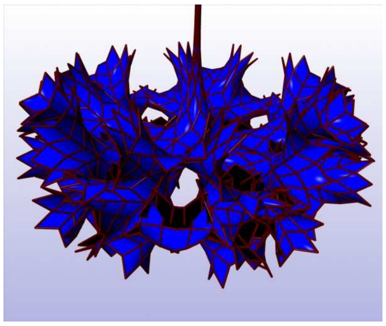

FIGURE 2. Henderson-Taimina-Thurston Discrete K-Surface

3. New Examples

There are very few examples of discrete K-surfaces with higher genus or several ends. Of particular interest are those which approximate K-surfaces. FIGURE 3 shows an example by Tim Hoffmann based on work in his dissertation [5]. By

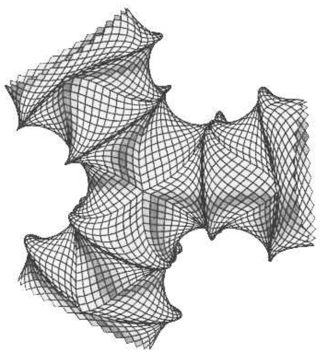

FIGURE 3. Hoffman Discrete K-Surface

adjusting the parameters in the Thurston construction it is possible to find other examples (FIGURE 4).

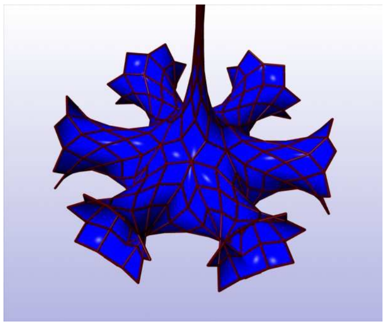

FIGURE 4. Hoffman-Sterling Discrete K-Surface

4. Computer Graphics

The computer graphics used here were generated using jReality. jReality is a Java 3D viewer for mathematics. A demonstration of some of its features were presented by Ulrich Pinkall. More details and examples can be found at www.jreality.de.

References

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