MATH 256 – EXAM 2

This exam is due, under my office door or in my hands by 3:00 PM on Friday, April 1 (No Joke!).
Directions: All work done on this exam must be your own. You are not allowed to seek help from the internet nor any person. You may ask me, Heather, anything you want, and I'll answer whatever I feel is reasonable to answer. All answers should be clearly stated in **complete sentences**. All work should be clean and clear. I will award points for clarity, accuracy of language, and correctness. Simple algebra mistakes that do not simplify the problem too much will not count against you. Have fun with this exam.

Sign Here stating that you did not receive help from any source but your notes, book, and Heather:

This page must be turned in with your exam.

(1) Given $F : V \to W$. Let $V = \left\{ \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \middle| \, a + 2b + 3c = 0, b - 4d = 0 \right\}$ define, for $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \in V$

$$F \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} d \\ c \\ f \\ e \end{pmatrix}$$

(a) Find a basis, $\mathcal{B}_1$ for the domain of F.
(b) Find a basis, $\mathcal{B}_2$ for the codomain of F.
(c) Find a basis for $\text{ran}(F)$.
(d) Find a basis for $\text{null}(F)$.
(e) Find $\text{nullity}(F)$.
(f) Find $\text{rank}(F)$.
(g) Is F injective? If not, find a counterexample. If so, prove it.
(h) Is F surjective? If not, find a counterexample. If so, prove it.

(2) Given $F : V \to W$. Let $V = \{ax^4 + bx^2 + cx + d | \, a + b + c = 0, b - d = 0\}$ define, for $(ax^4 + bx^2 + cx + d) \in V$

$$F\left(ax^4 + bx^2 + cx + d\right) = \begin{array}{|c|} \hline b \\ \hline 3b + 2d \\ \hline d \\ \hline \end{array}$$

(a) Find a basis, $\mathcal{B}_1$ for the domain of F.
(b) Find a basis, $\mathcal{B}_2$ for the codomain of F.
(c) Find the matrix representation, M, of F.
(d) Find a basis for $\text{col}(M)$.
(e) Find a basis for (M).
(f) Find $\text{nullity}(M)$.
(g) Find $\text{rank}(M)$.
(h) Is F injective? If not, find a counterexample. If so, prove it.
(i) Is F surjective? If not, find a counterexample. If so, prove it.

(3) Find a linear transformation $F : \mathcal{P}_3 \to \mathbb{R}^5$ that is onto. Prove your transformation is indeed onto or explain, clearly, why such a transformation cannot exist.