MATH 256 – EXAM 1–SPRING 2016

Directions: All work done on this exam must be your own. You are not allowed to seek help from the internet nor any person. You may ask me, Heather, anything you want, and I'll answer whatever I feel is reasonable to answer. All answers should be clearly stated in complete sentences. All work should be clean and clear. I will award points for clarity, accuracy of language, and correctness. Simple algebra mistakes that do not simplify the problem too much will not count against you. Have fun with this exam.

Sign Here stating that you did not receive help from any source but your notes, book, and Heather:

This page must be turned in with your exam.

(1) Determine whether or not

$$V = \{(a + 2b - 2)x^2 + 2(a + b)x - 4b | a, b \in \mathbb{R}\}$$

is a vector space. If so, prove it. If not, state the reason why not.

(2) Given the set $V = \{I | a, b, c, d \in \mathbb{R}\}$ where

$$I = \begin{array}{|c|c|c|} \hline & 3a & \\ \hline a - b & & a \\ \hline & b + a & \\ \hline 2b - c & & b \\ \hline & c + b & \\ \hline \end{array}$$

(a) Show V is a vector space.
(b) Find a basis for V.

(3) Find a basis for the vector space $V = \text{span}\{2x^2 - 5, x + 1, x - 1, 4\}$.

(4) Is $v = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ in the space spanned by the set of vectors $\left\{ \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$? Prove your answer.

(5) Determine whether the set $\left\{ \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$ is linearly independent or dependent. Prove your answer.

(6) Given a basis $\mathcal{B} = \{u, v, w\}$ for the vector space V, do the following:
(a) State the dimension of B.
(b) Determine whether or not $\mathcal{B}'\{u + v, u - v, w + 2u + 2v\}$ is also a basis for V. Prove your answer.
(c) Find a completely different basis for V. That is, find a basis that has no vector in common with $\mathcal{B}$ nor with $\mathcal{B}'$.

(7) Find a basis, $\mathcal{B}$ for the vector space

$$V = \{ax^3 + bx^2 + cx + d | a + b + c + d = 0\}.$$

(8) Use 7 to answer the following questions.
(a) Show that $u = -x^3 + 3x^2 - 4x + 2 \in V$?
(b) Find the coordinate vector $[u]_{\mathcal{B}}$.

(9) Write a description (more than just a sentence or two) of the various connections we've made so far in class relating the following topics:
- Solutions to systems of equations
- Solutions to homogeneous systems of equations
- Determinants
- Linear dependence/independence
- Span
- Basis