Radiography and Tomography in Linear Algebra
Lab #4

In this activity, you will explore some of the properties of radiographic transformations.

In Lab #3 you investigated the questions: (1) whether a radiographic operator can produce the same radiographs from different objects, (2) whether there are invisible objects other than the "zero" object, and (3) whether there are radiographs that are not the image of any object.

Task 1 After reviewing Lab 3, answer the following questions about spaces related to a radiographic transformation T.

1. For a radiographic transformation T, is the nullspace of T a subspace consisting of objects or of radiographs?
2. Describe the importance of the nullspace of a radiographic transformation given that the goal is to reconstruct objects.
3. For a radiographic transformation T, is the column space of T a subspace consisting of objects or of radiographs?
4. Describe the importance of the column space of a radiographic transformation given that the goal is to reconstruct objects.

Task 2 Now consider the questions of whether T is injective and/or surjective.

1. What does it mean for T to be injective?
2. What does it mean for T *not* to be injective? If two objects produce the same radiograph, what can you say about the 'object' that is their difference?
3. What does it mean for T to be surjective? Not surjective?

Task 3 Using your answers above, consider the following questions.

1. What must be true about the radiographic transformation T so that can we determine (reconstruct) the object that produces a given radiograph?
2. Suppose that the matrix corresponding to the radigraphic transformation T is injective and square. In order to reconstruct an object that produces a given radiograph, what must be true about the transformation?
3. Is it possible for the radiographic transformation to correspond to a nonsquare matrix that is taller than it is wide (more rows than columns)? If so, what would this mean? If not, why not?

4. Is it possible for the radiographic transformation to correspond to a nonsquare matrix that is wider than it is tall (more columns than rows)? If so, what would this mean? If not, why not?

5. Brainstorm ideas that might help to reconstruct an object for a given radiograph if the radiographic transformation is injective, but the corresponding matrix is not square.