Radiography and Tomography in Linear Algebra
Lab #3

In this activity, you will explore some of the properties of radiographic transformations.

In Lab #2 you found six radiographic transformation operators. The object image consisted of four voxels and was represented as a vector in $\mathbb{R}^4$. The radiographic image was represented as a vector in $\mathbb{R}^M$, one entry for each radiographic pixel. The price we pay for this representation is that we no longer have the geometry of the radiographic setup encoded in the representation. The use of representations in $\mathbb{R}^n$ is a computational tool and not geometrically descriptive of vector spaces of images. We want to reiterate that this is *only* a representation and that these images are **not** vectors in $\mathbb{R}^M$. Because of these new image representations, each transformation could be constructed as a matrix operator in $\mathcal{M}_{M\times 4}(R)$.

Task 1

For each of the radiographic transformations 1,3 and 4 which you found in Lab #2 answer the following questions. Justify your conclusions.

1. Is it possible for two different objects to produce the same radiograph? If so, give an example.
2. Are any nonzero objects invisible to this operator? If so, give an example. We say that an object is nonzero if not all entries are zero. We say that an object is invisible if it produces the zero radiograph.
3. Are there radiographs (in the appropriate dimension for the problem) that cannot be produced as the radiograph of *any* object? If so, give an example.

Task 2

Go a little deeper into understanding the operators 1, 3, and 4 from Lab #2 by answering these questions.

1. Choose any two objects that produce the same radiograph and subtract them. What is special about the resulting object?
2. Describe the set of all invisible objects. This could involve an equation that the entries would have to satisfy or a few specific objects that could be used to construct all other such objects. Be creative.
3. Describe the set of radiographs that can be produced from all possible objects. This may require similar creativity.

Task 3

Now put this more into the language of Linear Algebra. For each of the operators 1, 3, and 4 from Lab # 2 do the following.

1. Give a basis for the set of all invisible objects.

2. Give a basis for the set of all possible radiographs.