

Radiography for Linear Algebra Lab 2

Radiographic Scenarios and Notation

A single-view radiographic setup consists of an area of interest where the object will be placed, and a single screen onto which the radiograph will be recorded. A multiple-view radiographic setup consists of a single area of interest experimentally designed so that radiographs of this area can be recorded for different locations about the object.

The geometry of radiographic scenarios is illustrated in figures 1 and 2. The notation is as follows.

- Slice of region of interest: n by n array of voxels.
- Total number of voxels in each slice is $N = n^2$.
- Each voxel has a width and height of 1 unit.
- For each radiographic view we record m pixels of data.
- The width of each pixel is $ScaleFac$. If $ScaleFac = 1$ then pixel width is the same as voxel width.
- Number of radiographic angles (views): a .
- Total number of pixels in the radiograph: $M = am$
- Angle of the i^{th} view (measured in degrees east of south): θ_i
- Object mass at voxel j is x_j
- Recorded radiograph value at pixel k is b_k

In this Lab we will be constructing matrix representations of the radiographic transformation operators for example scenarios. Recall that an object can be represented by an N -dimensional vector x and a set of radiographs can be represented by an M -dimensional vector b . What will be the size of the corresponding matrix operator that maps object vector x to radiograph vector b ? Read again the definition of the matrix operator from the Radiography lecture notes. Notice that particular values for x and b are *not necessary* for computing the matrix of the radiographic transformation!

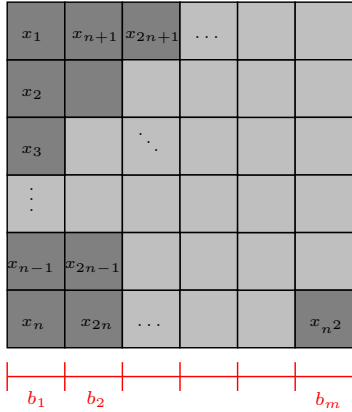


Figure 1: The geometry of a single view radiographic transformation.

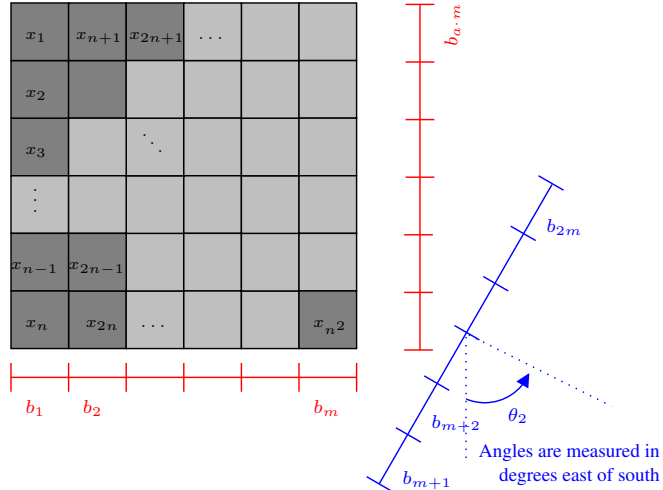
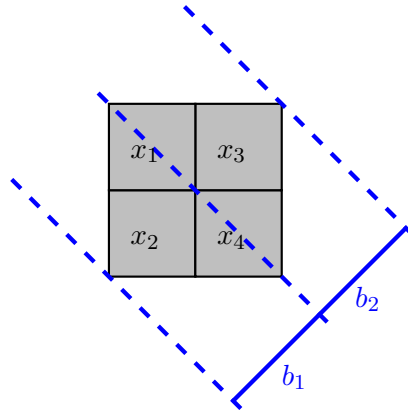


Figure 2: The Geometry of a multiple view radiographic transformation showing view 1, view 2, and view a .

A First Example

Let's look at a specific scenario. For the setup pictured below, we have:

- Total number of voxels: $N = 4$ ($n = 2$).
- Total number of pixels: $M = m = 2$
- $ScaleFac = \sqrt{2}$
- Number of views: $a = 1$
- Angle of the single view: $\theta_1 = 45^\circ$



Recalling that T_{kj} is the fraction of voxel j which projects perpendicularly onto pixel k , the matrix associated with this radiographic setup is $T = \begin{bmatrix} 1/2 & 1 & 0 & 1/2 \\ 1/2 & 0 & 1 & 1/2 \end{bmatrix}$. Be sure and check this to see if you agree! Hence, for *any* input vector x , the radiographic output is

$$b = Tx. \text{ Find the output when the object is the vector } x = \begin{bmatrix} 10 \\ 0 \\ 5 \\ 10 \end{bmatrix}.$$

Now it's your turn!

Note: Some of the following exercises will ask you to use Matlab or Octave to compute radiographic transformations. You will need to first download the function `tomomap.m` from the class website. If you do not yet have access to personal or department computing

resources, you can complete this assignment using online Octave in a web browser. Instructions are given at the end of this document.

Part I: Finding the matrix associated with a radiographic transformation, and using the matrix to determine radiographic outputs.

1. Suppose you have the setup where

- Height and width of image in voxels: $n = 2$ (Total voxels $N = 4$)
- Pixels per view in radiograph: $m = 2$
- $ScaleFac = 1$
- Number of views: $a = 2$
- Angle of the views: $\theta_1 = 0^\circ, \theta_2 = 90^\circ$

- (a) Sketch this setup.
- (b) Calculate the matrix associated with the setup.
- (c) Find the radiographs of the following objects.

4	3
6	1

0	1
1	0

2. Suppose you have the setup where

- Height and width of image in voxels: $n = 2$ (Total voxels $N = 4$)
- Pixels per view in radiograph: $m = 2$
- $ScaleFac = 1$
- Number of views: $a = 2$
- Angle of the views: $\theta_1 = 0^\circ, \theta_2 = 45^\circ$

- (a) Sketch this setup.
- (b) Calculate the matrix associated with the setup.
- (c) Repeat step (b) using the code `tomomap`.

3. Suppose you have the setup where

- Height and width of image in voxels: $n = 2$ (Total voxels $N = 4$)
- Pixels per view in radiograph: $m = 2$
- $ScaleFac = \sqrt{2}$
- Number of views: $a = 2$
- Angle of the views: $\theta_1 = 45^\circ, \theta_2 = 135^\circ$

- (a) Sketch this setup.

- (b) Calculate the matrix associated with the setup.
- (c) Repeat step (b) using the code `tomomap`.
4. Suppose you have the setup where
- Height and width of image in voxels: $n = 2$ (Total voxels $N = 4$)
 - Pixels per view in radiograph: $m = 4$
 - $ScaleFac = \sqrt{2}/2$
 - Number of views: $a = 1$
 - Angle of the views: $\theta_1 = 45^\circ$
- (a) Sketch this setup.
- (b) Calculate the matrix associated with the setup.
- (c) Repeat step (b) using the code `tomomap`.
5. Suppose you have the setup where
- Height and width of image in voxels: $n = 4$ (Total voxels $N = 16$)
 - Pixels per view in radiograph: $m = 2$
 - $ScaleFac = 1$
 - Number of views: $a = 2$
 - Angle of the views: $\theta_1 = 0^\circ$, $\theta_2 = 45^\circ$
- (a) Sketch this setup.
- (b) Calculate the matrix associated with the setup.
- (c) Find the radiographs of images A, B, and C from lab 1 under this transformation.
- (d) Repeat steps (b) and (c) using the code `tomomap`.
6. Suppose you have the setup where
- Height and width of image in voxels: $n = 4$ (Total voxels $N = 16$)
 - Pixels per view in radiograph: $m = 4$
 - $ScaleFac = 1$
 - Number of views: $a = 3$
 - Angle of the views: $\theta_1 = 0^\circ$, $\theta_2 = 25.5^\circ$, and $\theta_3 = 90^\circ$
- (a) Sketch this setup.
- (b) Calculate the matrix associated with the setup using `tomomap`.
- (c) Find the radiographs of images A, B, and C from lab 1 under this transformation.

Part II: Radiographs of Linear Combinations of Objects

Take the two objects in exercise 1 above to be x (left object) and y (right object). For each of the transformations in exercises 1, 3, and 4 from Part I answer the following questions.

1. What is the radiograph of the object $3x$?
2. What is the radiograph of the object $0.5y$?
3. What is the radiograph of the object $3x + 0.5y$?

4. Generalize these observations to arbitrary linear combinations of object vectors. Write your conjecture(s) in careful mathematical notation.

Instructions for using octave-online.net

1. Download the file `tomomap.m` from the class website to your computer in a location of your choice.
2. Open `octave-online.net` in a web browser. Close (or read and close) the introduction splash window.
3. Click the three bars in the upper right of the screen and sign in.
4. The far left window is a list of your files. You need to place the file `tomomap.m` into this octave-online folder. To do this either (a) drag and drop `tomomap.m` onto this window or (b) click the upload file icon in the window, find `tomomap.m`, and click open. The file opens automatically in the editor window.
5. FYI: The rightmost window is the command prompt terminal. You will type your commands here. You also have the option of creating your own scripts and functions to save and run.
6. Run this example call to function `tomomap.m` by typing the following at the command prompt and hitting return.

```
T = tomomap(2,2,[20 60],1)
```

You should see twelve entries in the variable T . All other entries are zero. To see the full array try the following:

```
T = full(tomomap(2,2,[20,60],1))
```

7. Now you are ready to complete the assignment. You can always read the comments at the beginning of `tomomap.m` (see editor window) to understand more about what this code does and how to use it.