MATH 256 – HOMEWORK 6

(1) Determine whether the following sets form a basis for $\mathcal{P}_3$. If not, state your reason. If so, prove it.
   (a) $\{1, x, x^2 + x, 1 + x + x^3\}$
   (b) $\{x - 2, 2x^{-}2, x^3 - 2\}$
   (c) $\{1 + x + x^2 + x^3, 5 + 3x + 3x^2 + x^3, x^2, 3 + 2x + 2x^2 + x^3\}$

(2) Suppose $\mathcal{B} = \{v_1, v_2, v_3, v_4\}$ is a basis for V. Determine whether each of the following sets is a basis for V as well. If not, state your reason. If so, prove it.
   (a) $\{v_1 + v_3, v_2 + v_4\}$
   (b) $\{v_1, v_2, v_3, v_4, v_1 - 2v_3\}$
   (c) $\{2v_1 - 3v_3, v_3, 3v_1 + 4v_4 + v_3, v_3 + v_1 + v_4\}$
   (d) $\{2v_1 - 3v_3, v_2, 3v_1 + 4v_4 + v_3, v_3 + v_1 + v_4\}$

(3) Do the following matrix multiplies if possible. If it is not possible, state the reason why not.
   (a) $\begin{pmatrix} 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 \\ 3 & 8 \end{pmatrix}$
   (c) $\begin{pmatrix} -1 & 0 & 2 \\ 3 & 5 & 1 \\ 0 & 2 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ -4 & 0 \end{pmatrix}$
   (d) $\begin{pmatrix} 1 & 2 & 0 & 3 \\ 2 & 5 & -1 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ -1 & 6 \end{pmatrix}$

(4) Multiply: $\begin{pmatrix} a & b & c \\ d & f & g \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ Write your answer as a linear combination of the vectors: $\left\{ \begin{pmatrix} a \\ d \end{pmatrix}, \begin{pmatrix} b \\ f \end{pmatrix}, \begin{pmatrix} c \\ g \end{pmatrix} \right\}$.

(5) Use the last question to give a different way of multiplying matrices (different than what was discussed in class).