MATH 256 – HOMEWORK 5

(1) Determine whether the following sets are linearly independent.
  (a) $\{1, x, x^2\}$
  (b) $\{1, x + x^2, x^2\}$
  (c) $\{x^2 - 1, 1 + x, 1 - x\}$
  (d) $\{1, 1 - x, 1 + x, 1 + x^2\}$
  (e) $\{1, 2x, x - 1, 1 + x - 2x^2\}$
  (f) Which of the above is a basis for $\mathcal{P}_2$?

(2) Suppose $\{v_1, v_2, v_3, v_4\}$ are linearly independent. Determine if the following sets are linearly independent. Justify your answer. If not, remove only enough vectors to make the set independent.
  (a) $\{v_1, v_2\}$
  (b) $\{v_1, v_2, v_3, v_4, v_1 - 2v_3\}$
  (c) $\{v_1 + v_2, v_3, v_4\}$
  (d) $\{v_1 + v_3, v_2 + v_4, v_3, v_4\}$
  (e) $\{v_1 + v_2 + v_3 + v_4, v_1 - v_2 + v_3 - v_4, v_1 - v_2 - v_3 + v_4, v_1 - v_2 - v_3 - v_4\}$
  (f) $\{v_1 - 2v_2, v_2, v_3 - v_4 - v_2, v_4\}$
  (g) $\{v_1 - v_2, v_1 + v_2, 2v_1 + v_2 - v_3, v_1 - v_2 - v_3 - 2v_4, v_3 - v_4\}$
  (h) Which of the above is a basis for $\text{span}\{v_1, v_2, v_3, v_4\}$?

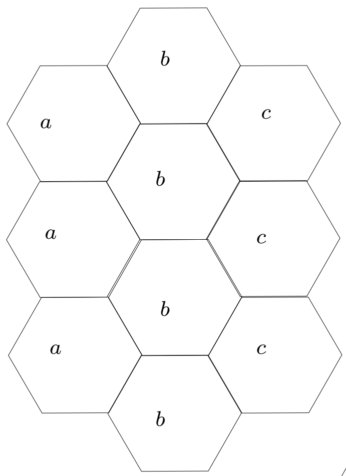
(3) For each of the following decide whether or not $\mathcal{B}$ is a basis for the vector space V. Justify your answer by showing that either all the properties of a basis are true or that one is false.
  (a) $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$, $V = \mathbb{R}^3$
  (b) $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$, $V = \mathbb{R}^2$
  (c) $\mathcal{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$, $V = \mathcal{M}_{2\times 2}$
  (d) $\mathcal{B} = \{x^2, x^2 + x, x^2 + x + 2\}$, $V = \mathcal{P}_2$

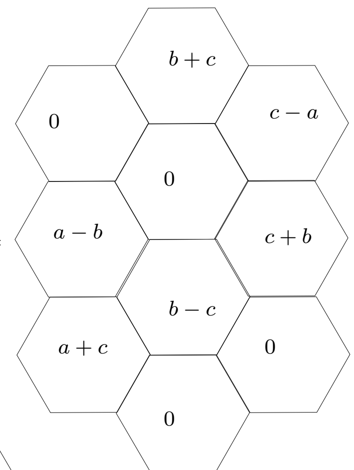
(4) For each of the vector spaces below, find basis $\mathcal{B}$ that is not the standard basis nor is it a basis on this sheet already.
  (a) $\left\{ \begin{pmatrix} a & c \\ 3d & b \end{pmatrix} \middle| \, a + b + c - 2d = 0, a + 3b - 4c + d = 0, a - d + b = c \right\}$
  (b) $\{cx^2 + 3bx - 4a | \, a - b - 2c = 0\}$
  (c) $\mathcal{M}_{2\times 2}$
  (d) $\mathcal{P}_2$
  (e) $\text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

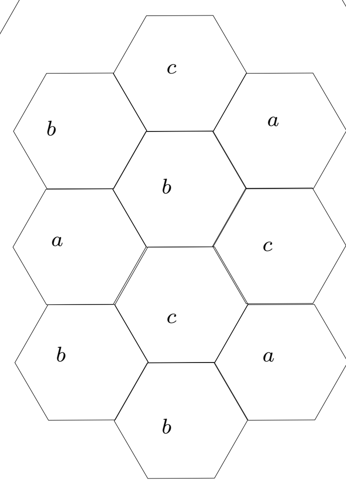
(5) Given the set $\mathcal{B} = \{u, v, w\}$. Show that if $\mathcal{B}$ is a basis, then so is $\mathcal{B}' = \{u + 2v, u - w, v + w\}$.

(6) Using #5, make a general statement about how to get a basis from another basis. Be careful to use accurate linear algebra language.

(7) Determine whether $\mathcal{B} = \{I_1, I_2, I_3\}$, where the I_n are given below, is a basis for the vector space of images of the same as the I_n orientation. Justify your answer by showing all properties of basis are true or by showing one is not.

$I_1 =$ 

$I_2 =$ 

$I_3 =$ 

(8) Determine the dimension of each of the above vector spaces.