MATH 256 – HOMEWORK 4

(1) Which of these sets is a vector space? Prove your answer.

(a) $\left\{ \begin{pmatrix} a & 1 \\ 2 & b \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\}$

(b) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| \, a + b = 0, a - b = 2 \right\}$

(c) $\left\{ \begin{pmatrix} a & 0 \\ c & b \end{pmatrix} \middle| \, a + b = c \right\}$

(d) $\left\{ \begin{pmatrix} a & c \\ 3d & b \end{pmatrix} \middle| \, a + b + c - 2d = 0, a + 3b - 4c + d = 0, a - d + b = c \right\}$

(2) Which of these sets is a vector space? Prove your answer.

(a) $\{x^2 + 3bx - 4a | \, a, b \in \mathbb{R}\}$

(b) $\{cx^2 + 3bx - 4a | \, a + b - c = 2\}$

(c) $\{cx^2 + 3bx - 4a | \, a - b - 2c = 0\}$

(3) Given the set {◎, ◈, ⧉, ☆}. Which of the definitions for + and scalar multiplication makes this set is a vector space? Prove your answer.

(a) + is defined as in the table and scalar multiplication is by integers in the usual sense:

+	◎	◈	⧉	☆
◎	◎	◎	◎	◎
◈	◎	⧉	◈	☆
⧉	◎	◈	☆	⧉
☆	◎	☆	⧉	◈

(b) where + is defined as in the table and scalar multiplication is by integers in the usual sense:

+	◎	◈	⧉	☆
◎	◈	◎	☆	⧉
◈	◎	◈	⧉	☆
⧉	☆	⧉	◈	◎
☆	⧉	☆	◎	◈

(c) where + is defined as in the table and scalar multiplication is by integers in the usual sense:

+	◎	◈	⧉	☆
◎	⧉	⧉	⧉	⧉
◈	⧉	◈	◈	◈
⧉	⧉	◈	☆	☆
☆	⧉	◈	☆	◎

(4) For each of the vector spaces in questions 1 and 2, write each as a span.

(5) For each of the vector spaces in questions 1 and 2, write each as a span that is different than the span you wrote in question 4.

(6) For each of the vector spaces in questions 1 and 2, write each as a span of two more vectors than the set chosen in question 5.

(7) Find a basis for each of the vector spaces in questions 1 and 2.
(8) Find a different basis than the one you found in question 7 for each of the vector spaces in questions 1 and 2.
(9) Find a basis for $\{\, I \,|\, a, b, c, d \in \mathbb{R}$ and I is the image below$\}$