MATH 256 – HOMEWORK 3

(1) Which of these subsets are subspaces of $\mathcal{M}_{2\times 2}$? For each one that is a subspace, write the set as a span. For each that is not, show the condition that fails.

(a) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| a, b \in \mathbb{R} \right\}$

(b) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| a + b = 0 \right\}$

(c) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| a + b = 5 \right\}$

(d) $\left\{ \begin{pmatrix} a & c \\ 0 & b \end{pmatrix} \middle| a + b = 0, c \in \mathbb{R} \right\}$

(2) Is this a subspace of $\mathcal{P}_2$?

$$\{a_0 + a_1 x + a_2 x^2 | a_0 + 2a_1 + a_2 = 4\}$$

If it is, then write it as a span. If not, change the set only a little to make it a subspace.

(3) Decide if the vector lies in the span of the set. If it does, find the linear combination that makes the vector. If it does not, show that no linear combination exists.

(a) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ in $\mathbb{R}^2$.

(b) $x - x^3$, $\{x^2, 2x + x^2, x + x^3\}$, in $\mathcal{P}_3$

(c) $\begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix}$, $\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix} \right\}$, in $\mathcal{M}_{2\times 2}$

(4) Which of these sets spans $\mathbb{R}^3$?

(a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(c) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \right\}$

(d) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \right\}$

(e) $\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix} \right\}$

(5) Find a set to span the given subspace

(a) The xz-plane in $\mathbb{R}^3$

(b) $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \, 3x + 2y + z = 0 \right\}$ in $\mathbb{R}^3$

(c) $\left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \, 2x + y + w = 0 \text{ and } y + 2z = 0 \right\}$ in $\mathbb{R}^4$

(d) $\{a_0 + a_1x + a_2x^2 + a_3x^3 | a_0 + a_1 = 0 \text{ and } a_2 - a_3 = 0\}$ in $\mathcal{P}_3$

(e) The set $\mathcal{P}_4$ in the space $\mathcal{P}_4$

(f) $\mathcal{M}_{2\times 2}$ in $\mathcal{M}_{2\times 2}$

(6) Is $\mathbb{R}^2$ a subspace of $\mathbb{R}^3$? If yes, show it. If no, why not?

(7) Answer the following questions with justification.

(a) If $S \subset T$ are subsets of a vector space is span$S \subset$ span$T$? Always? Sometimes? Never?

(b) If S, T are subsets of a vector space is span$(S \cup T) =$ span$S \cup$ span$T$?

(c) If S, T are subsets of a vector space is span$(S \cap T) =$ span$S \cap$ span$T$?

(d) Is the span of the complement equal to the complement of the span?