

## MATH 256 – Homework 13

(1) For each of the following matrices, find the eigenvalues and the corresponding eigenspaces.

(a)  $\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$

(b)  $\begin{pmatrix} -1 & 1 \\ -3 & 3 \end{pmatrix}$

(c)  $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

(d)  $\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 2 & -2 & 1 \end{pmatrix}$

(e)  $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(f)  $\begin{pmatrix} 6 & 10 & -4 \\ -2 & -2 & 2 \\ -2 & 2 & 0 \end{pmatrix}$

(g)  $\begin{pmatrix} 0 & -4 & 1 \\ .5 & 3 & -.5 \\ 0 & 0 & 1 \end{pmatrix}$

(h)  $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$

(i)  $\begin{pmatrix} 2 & -3 & -1 \\ 8 & -3 & -7 \\ -1 & -3 & 2 \end{pmatrix}$

(j)  $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(2) Find a matrix with the given eigenvalues and eigenspaces.

(a)  $\lambda_1 = 1, \lambda_2 = 2$  corresponding to eigenspaces

$$\mathcal{E}_1 = \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\} \quad \text{and} \quad \mathcal{E}_2 = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

(b)  $\lambda_1 = 1, \lambda_2 = 0$  corresponding to eigenspaces

$$\mathcal{E}_1 = \text{span} \left\{ \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\} \quad \text{and} \quad \mathcal{E}_2 = \text{span} \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$$

(c)  $\lambda_1 = 1, \lambda_2 = -1$  corresponding to eigenspaces

$$\mathcal{E}_1 = \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\} \quad \text{and} \quad \mathcal{E}_2 = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

(3) Diagonalize each of the matrices in exercise (1). If it is not possible, state why.