MATH 256 – Homework 10

(1) Determine which of the following is a linear transformation. Prove your answer.

(a) $f(v) = Mv + x$, where

$$M = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

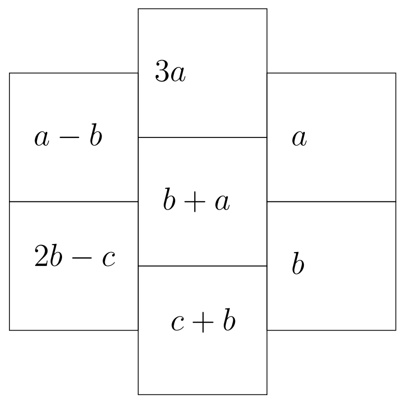
(b) $\mathcal{F}(ax^2 + (3a - 2b)x + b) = 2ax + 3a - 2b$

(c) $\mathcal{G}(ax^2 + bx + c) = \begin{pmatrix} a & a-b \\ c-2 & c+3a \end{pmatrix}$

(d) $h\begin{pmatrix} a & b & c \\ 0 & b-c & 2a \end{pmatrix} = ax + c$

(e) $f(I) = ax^2 + (b+c)x + (a+c)$, where I is the image below

$I =$



(f) $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

(g) $f(ax^2 + bx + c) = \begin{pmatrix} a+b \\ a-c \end{pmatrix}$

(2) Find the nullspace and nullity of each of the transformations from exercise 1. Be sure to clearly label which is the nullspace and which is the nullity.

(3) For each of the transformations in exercise 1, state the domain, V, codomain W, $\dim(W)$, $\dim(V)$, $\text{rank}(T)$ and verify the Rank-nullity Theorem using your answer from exercise 2.

(4) For each of the domain spaces, V, specified in exercise 3, find a isomorphism $f : V \to \mathbb{R}^d$, where $d = \dim(V)$.

(5) For each of the codomain spaces, W, specified in exercise 3, find an isomorphism $g : \mathbb{R}^w \to W$ where $w = \dim(W)$.

(6) For each of the transformations in exercise 1, find the associated matrix.

(7) Determine which of the transformations in exercise 1 are injective. Prove your answer.

(8) Determine which of the transformations in exercise 1 are surjective. Prove your answer.

(9) For each of the domain spaces V found in exercise 3, find two bases $\mathcal{B}_1$ and $\mathcal{B}_2$ with no common vector. (If $\mathcal{B}_1 = \{a, b, c\}$ then $\mathcal{B}_1$ cannot contain, a, b, nor c.) Then, find the change of basis matrix to get from $\mathcal{B}_1$ to $\mathcal{B}_2$.

Added problems next page:

For each of the transformations:

A. T_3 the third radiographic transformation from Lab 2.-
B. $\frac{d}{dx} : \mathcal{P}_4 \to \mathcal{P}_3$
C. $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$, where θ is a specific fixed angle measured from the x axis and R_θ is the rotation transformation
D. $\pi_v : \mathbb{R}^3 \to \mathbb{R}^3$, the component of v parallel to the x axis.
E. $D : \mathbb{R}^3 \to \mathbb{R}$ defined by $D(v) = v \cdot u$, where $u = (0, 1, -1)$.

Answer each of the following:

(1) What is null(T)?
(2) What is ran(T)?
(3) Is the transformation 1-1? Prove it.
(4) Is the transformation onto? Prove it.