name:____________________________________

MATH 256 – EXAM 1

Directions: You may use *your own* textbook but **not** any solutions manual. You may use **your own** notes and **your own** homework. You may not use the internet or any other electronic or printed resource including calculators, mathematical software, or other books. You may not use anyone elses notes. If you work on this exam in a public space you must keep your work private. Do not leave your work where it is visible. Erase boards if you work on them. You may not discuss this exam with or near any person except Dr. Heather Moon. Failure to follow the letter and spirit of these instructions will result in you failing the course. Your solutions should be **written very clearly and clean**. Your audience is not Heather, nor is it Morgan. Write something your classmates will understand. You must provide sufficient reasoning to completely convince any of your classmates. The exam is due at 3:00 PM on Friday, November 13.

(1) Prove whether or not the following is a linear transformation, $F : V \to W$.

  (a) Let $V = \left\{ \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \middle| \, a + 2b + 3c = 0, b - 4d = 0 \right\}$ define $F \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = a - b$

  (b) $F(ax + b) = \begin{pmatrix} a & b + a \\ a - b & 3a \end{pmatrix}$

(2) For each of the linear transformations in Problem 1, find $\text{null}(F)$, $\text{ran}(F)$, $\text{nullity}(F)$, and $\text{rank}(F)$.

(3) Show that if $T : V \to W$ is a linear transformation that $\text{null}(T)$ is a subspace of V.

(4) Show that if $T : V \to W$ is a linear transformation that $\text{ran}(T)$ is a subspace of W.

(5) Find a linear transformation $F : \mathcal{P}_1 \to \mathbb{R}^2$ that is not an isomorphism, but is one-to-one. Prove your result. That is, prove F is one-to-one and prove that F is not an isomorphism.

(6) Find a linear transformation $F : \mathcal{M}_{2\times 2} \to \mathcal{P}_2$ that is onto, but not an isomorphism. Prove your result. That is, prove F is onto and prove that F is not an isomorphism.

(7) Find a linear transformation $F : \mathcal{P}_3 \to \mathbb{R}^5$ that is onto or explain, clearly, why this cannot exist.