Heat Equation for Linear Algebra Lab 2

Recall that our goal is to find the heat state after k time steps in the heat diffusion. We have observed that some heat states (simple vectors) only change in amplitude with time, $Ev_j = a_j v_j$. Now we will explore what could happen if some set of simple vectors $\beta = \{v_1, v_2, \cdots, v_m\}$ is a basis for $\mathbb{R}^m$.

1. Working with the standard basis for $\mathbb{R}^m$, what can you determine about the diffusion of a heat state $u(t + \Delta t) = Eu(t)$ given basis β?

2. Use your answer from question 1 to find a representation for the heat state after the 10th iteration of diffusion, that is, for $u(t + 10\Delta t)$.

3. Answer question 1 again, but consider a change of basis.

4. Using your answer from question 3, find a representation for the heat state after the 10th iteration of the diffusion.

5. How might these computations using the simple vector basis give us information about the long term behavior of the heat diffusion? Make some observations.