Heat Equation for Linear Algebra – Homework 1

In class, we found the matrix representation of the heat diffusion operator to be

$$E = \begin{bmatrix} 1-2\delta & \delta & 0 & 0 & \\ \delta & 1-2\delta & \delta & 0 & \dots \\ 0 & \delta & 1-2\delta & \delta & \\ & & \vdots & & \ddots \end{bmatrix}.$$

We see that if we are want to find the kth heat state, $u(k\Delta t)$ in the heat diffusion, we need to use

$$u(k\Delta t) = E^k u(0),$$

where $u(0)$ is the initial heat state. Let's explore what happens when we try to do this.

(1.) Find E^2.

(2.) Does it look like raising E to higher and higher powers is an easy way to find the heat state at some time far in the future?

(3.) Pick your favorite nontrivial vector $u \in \mathbb{R}^4$ and compute Eu, $E(Eu)$ and $E(E(Eu))$.

(4.) Does it look like multiplying over and over by the matrix E is an easy way to find the heat state at some time far in the future?

(5.) If you had to perform this kind of operation, what characteristics would you prefer a diffusion operator to have?