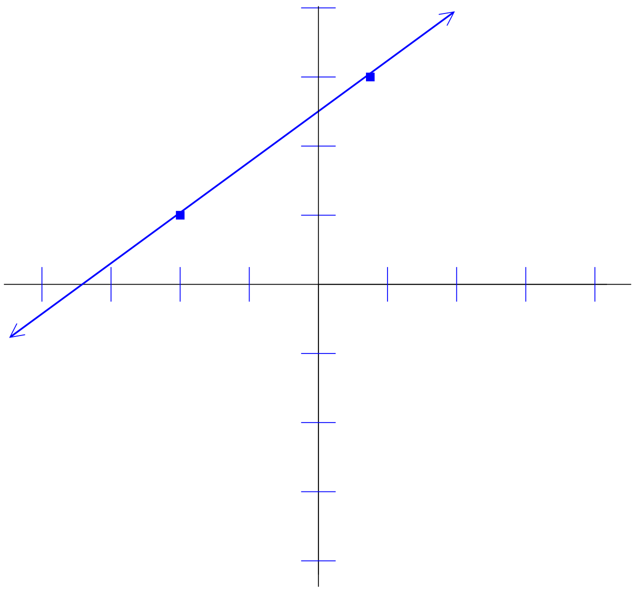
**Precalculus review Solutions–Wednesday Evening, January 20, 2016**

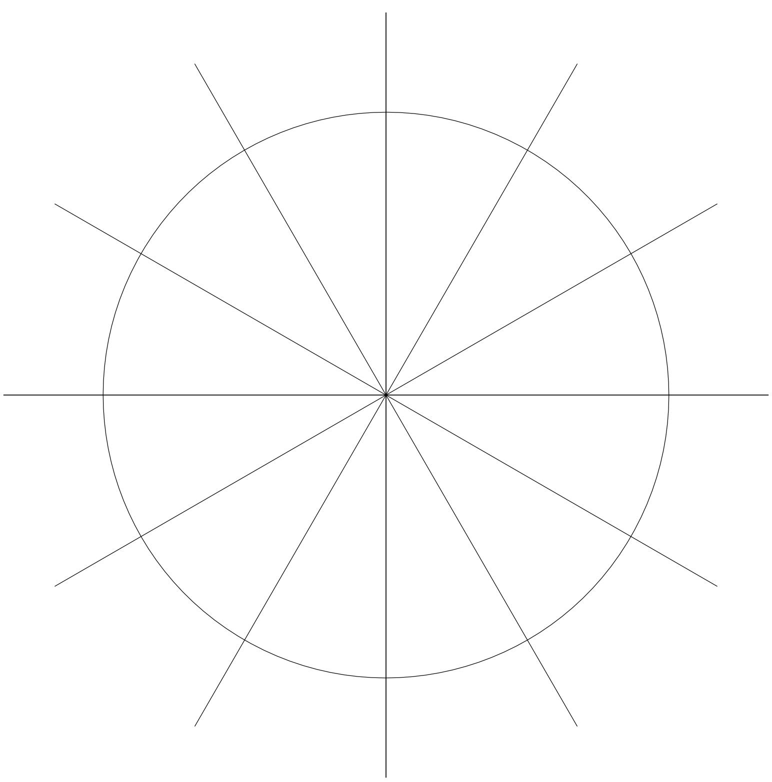
1. Find the equation of the line with slope $m = \frac{2}{3}$ and passing through $(1, 3)$. Then, find the intercepts of this line. Draw the line below. (Be sure to label your axes accurately. Don't make 1 too small.)



Equation:
$y - 3 = \frac{2}{3}(x - 1)$

$y$-intercept:
$y - 3 = \frac{2}{3}(0 - 1)$
$y = \frac{2}{3} + 3 = \frac{7}{3}$
$(0, \frac{7}{3})$
$x$-intercept:
$0 - 3 = \frac{2}{3}(x - 1)$
$-3 = \frac{2}{3}(x - 1)$
$-9 = 2x - 2$
$-7 = 2x$
$x = -\frac{7}{2}$
$(-\frac{7}{2}, 0)$

2. Completely, fill in the unit circle below.



If you didn't get this completed, be sure to figure it out. Ask me. Ask Sarah.

3. Use the unit circle and trigonometric identities to find $\sin(\frac{\pi}{12})$.

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

4. Solve the equation $3x^2 + 4x + 1 = 0$ in two ways: Using the quadratic formula and factoring. (Be sure you get the same answer.)

Factoring

$$\begin{aligned} 3x^2 + 4x + 1 &= 0 \\ (3x+1)(x+1) &= 0 \\ x = -\tfrac{1}{3} \quad & x = -1 \end{aligned}$$

Quadratic Formula

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4(3)(1)}}{2(3)} \\ &= \frac{-4 \pm \sqrt{16-12}}{6} \\ &= \frac{-4 \pm \sqrt{4}}{6} \\ &= \frac{-4 \pm 2}{6} \\ \text{x} &= -\frac{1}{3}, -1 \end{aligned}$$

5. Solve the equation $2x^2 + 8x + 1$ by completing the square.
This is not an equation so it cannot be solved. But that was a typo on my part. I meant for it to say $2x^2 + 8x + 1 = 0$. Anyway, you can still complete the square:

$$2x^{+}8x + 1 = 2(x^2 + 4x) + 1 = 2(x^2 + 4x + 4 - 4) + 1 = 2(x+2)^2 - 8 + 1 = 2(x+2)^2 - 7.$$

6. Write the equation of a circle with center $(1, 1)$ and radius $r = 3$.
$(x-1)^2 + (y-1)^2 = 9$

7. Find the equation of the parabola passing through the three points: $(1, 0)$, and $(2, 5)$, and $(-1, 8)$.
First, we write down the general equation for a parabola:

$$y = ax^2 + bx + c.$$

Now we can write down the equations that come out of this with the above points:

$$\begin{aligned} (1,0): &\quad 0 = a + b + c \\ (2.5): &\quad 5 = 4a + 2b + c \\ (-1,8): &\quad 8 = a - b + c \end{aligned}$$

We can solve for $a, b$ and $c$ using substitution or elimination. I prefer elimination:
Subtract the first equation minus the third to get

$$2b = -8 \quad \Rightarrow \quad b = -4.$$

Put this into the first and second equations and we get

$$\begin{aligned} 0 &= a - 4 + c \\ 5 &= 4a - 8 + c \end{aligned}$$

Now, we can subtract these two equations and we get:

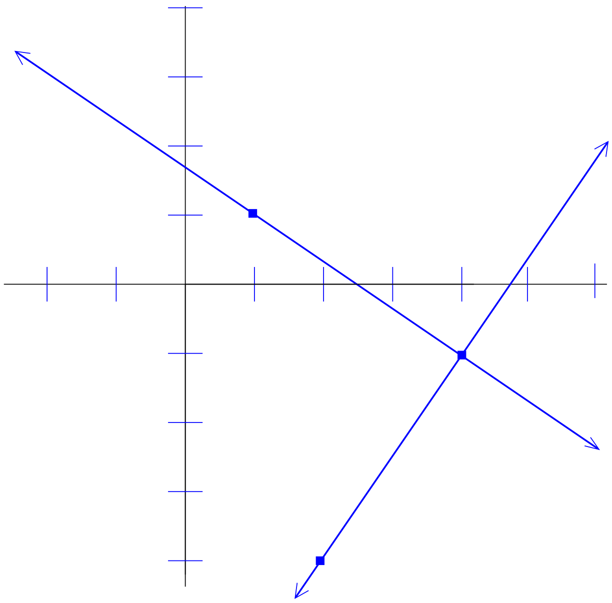
$$5 = 3a - 4 \quad \Rightarrow \quad 3a = 8 \quad \Rightarrow \quad a = 3.$$

Now put this back into one of the last two equations (I choose the first of these) and we get

$$0 = 3 - 4 + c \quad \Rightarrow \quad c = 1.$$

Thus, the parabola is $y = 3x^2 - 4x + 1$. Note: There are many ways to do this.

8. Find the point where the two equations cross: $2x + 3y = 5$ and $3x - 2y = 14$. Draw the equations on the axes provided to show that your solution is correct.



Equation:
$2x + 3y = 5$
$y = -\frac{2}{3}x + \frac{5}{3}$
Slope and $y$-intercept:
$m = -\frac{2}{3}, y = \frac{5}{3}$

$3x - 2y = 14$
$y = \frac{3}{2}x - 7$
Slope and $y$-intercept
$m = \frac{3}{2}, y = -7$

Where they cross:
$-\frac{2}{3}x + \frac{5}{3} = \frac{3}{2}x - 7$
$\frac{26}{3} = \frac{13}{6}x$
$x = 4 \Rightarrow y = -1$

9. Factor the following polynomials completely:

(a) $8x^3 - 27$
$(2x - 3)(4x^2 + 6x + 9)$

(b) $4x^2 - 9$
$(2x - 3)(2x + 3)$

(c) $3x^4 - 6x^3 - 9x^2$
$3x^2(x^2 - 2x - 3) = 3x^2(x - 3)(x + 1)$

(d) $x^4 - 64$
$(x^2 - 8)(x^2 + 8)$

(e) $3x^3 - 3x^2 + 4x - 4$
$3x^2(x - 1) + 4(x - 1) = (x - 1)(3x^2 + 4)$

10. Solve or simplify (state the difference before beginning any of these). The difference between solving and simplifying is that we solve equations (there's an equal sign) and we simplify expressions (there is no equal sign).

(a) $\dfrac{x - 2}{x + 4} = 3$
We will solve this since it is an equation.

$$\begin{aligned} \frac{x-2}{x+4} &= 3 \\ \Rightarrow x - 2 &= 3(x + 4) \\ \Rightarrow x - 2 &= 3x + 12 \\ 2x &= -14 \Rightarrow x = -7 \end{aligned}$$

(b) $\dfrac{\frac{2}{x+h}-\frac{2}{x}}{h}$ This is an expression so we simplify it.

$$\begin{aligned}\frac{\frac{2}{x+h}-\frac{2}{x}}{h} &= \frac{\frac{2}{x+h}-\frac{2}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}\\ &= \frac{2x-2(x+h)}{hx(x+h)}\\ &= \frac{2h}{hx(x+h)} = \frac{2}{x(x+h)}, h \neq 0\end{aligned}$$

11. Rationalize the denominator or numerator (whichever begins with radicals)

(a) $\dfrac{3}{\sqrt{2}}$

$\dfrac{3}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$

(b) $\dfrac{x}{\sqrt{2x+1}-\sqrt{x}}$

$\dfrac{x}{\sqrt{2x+1}-\sqrt{x}}\cdot\dfrac{\sqrt{2x+1}+\sqrt{x}}{\sqrt{2x+1}+\sqrt{x}} = \dfrac{\sqrt{2x+1}+\sqrt{x}}{2x+1-x} = \dfrac{\sqrt{2x+1}+\sqrt{x}}{x+1}$

(c) $\sqrt{x+1}-\sqrt{2x-1}$

$\sqrt{x+1}-\sqrt{2x-1}\cdot\dfrac{\sqrt{x+1}+\sqrt{2x-1}}{\sqrt{x+1}+\sqrt{2x-1}} = \dfrac{x+1-(2x-1)}{\sqrt{x+1}+\sqrt{2x-1}} = \dfrac{x+2}{\sqrt{x+1}+\sqrt{2x-1}}$