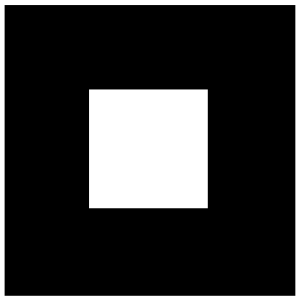
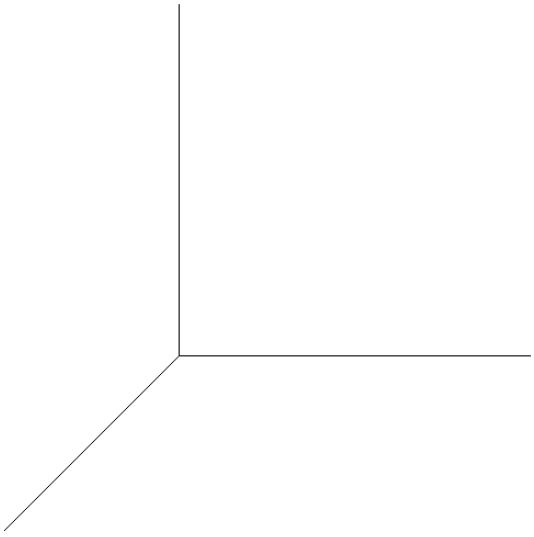
LAB #3. IMAGE DENOISING IN ANALYSIS: MEASURING NOISE.

1. Identifying the noise

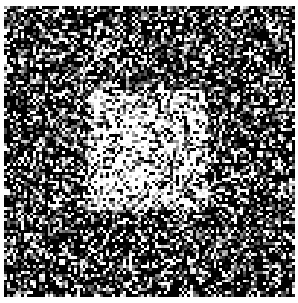
Last semester, we discussed noise on a function (and in an image). We saw that a function is noisy if there is a lot of variation (some called it "wiggles" or "jaggedness"). Let's use this information to explore the problem of denoising.

(1.1) On the axes provided draw the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ that represents the image below. (Assume pixels that are white have a pixel value of 1 and pixels that are black have a pixel value of 0.)





(1.2) Describe in words what the function would look like if the image was noisy (like the image below). (Note: I'm not asking you to draw this because it would be quite hard to draw such a function, continuous or not).



(1.3) Suggest one way you could measure how much the image varies around one particular pixel.

(1.4) Using your answer in Exercise 1.3, how could you measure the total variation in an image?

(1.5) Now, what if you happen to have the function that represents the image, how can you represent the total variation using mathematical ideas that might measure total variation (or total absolute change)? You can describe this in words or write a formula, we will discuss this as a class.

2. Suppose there's no noise...Is that enough?

(2.6) Both Image B and Image C below are noiseless images, but it is clear that Image C is the result of denoising image A. In words, describe how we know that Image C is a denoised version of Image A and Image B is not.

Image A

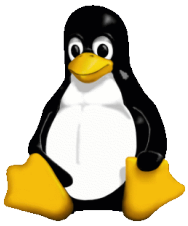


Image B

Image C

(2.7) Using your ideas form Exercise 2.6, suggest a way to measure whether the denoised image corresponds to the noisy image. (Again, assume you know the functions representing your noisy image and your "denoised" image.) You need not come up with a metric, instead, you should be able to describe using mathematical ideas.

3. Putting Things Together

(3.8) Based on your responses in the previous two sections, list the properties that we expect to see in a denoised image.

Before we write down the answer to the next question, we will discuss what we've learned so far.

(3.9) (a) Write down what your functional would look like based on your properties listed in Exercise 3.8.

(b) For posterity, write down the functional that we will use in this to denoise images (discussed in class).

(c) List the similarities and differences between your functional and the one we will use in this class.

(3.10) How does the denoised image relate to the functionals above?

In upcoming classes, we will introduce analysis terminology to describe functionals like the ones above. We will also develop some standard analysis tools that will be useful for us in denoising an image.