MATH 352: HOMEWORK 9
DUE TUESDAY APRIL 26

(1) Let $f : [0, 1] \to \mathbb{R}$ be the function $f(x) = 2x - 4$. Choose a partition of $[0, 1]$. Find $U(f, P)$, where P is the partition you chose.

(2) Let $f : [a, b] \to \mathbb{R}$ be a bounded function. Let $\mathcal{P}$ be an arbitrary partition of the domain of f. Show $U(f) \geq L(f, P)$.

(3) Prove or disprove: A bounded function f is integrable on $[a, b]$ if and only if, for every $\varepsilon > 0$, there exists a partition a partition $\tilde{P}$ of $[a, b]$ so that

$$U(f, \tilde{P}) - L(f, \tilde{P}) < \varepsilon.$$

(4) Let $f : [a, b] \to \mathbb{R}$ be an increasing function. Show that f is integrable.

(5) Consider $f(x) = 3x - 2$ over the interval $[1, 4]$. Let P be the partition consisting of the points $\{1, 2, 3, 4\}$.
   (a) Compute $L(f, P)$, $U(f, P)$, and $U(f, P) - L(f, P)$.
   (b) What happens to the value of $U(f, P) - L(f, P)$ when we add the point 1.5 to the partition?
   (c) Find a partition P' of $[1, 4]$ for which $U(f, P') - L(f, P') < 3$.

(MGC#3) Prove that f in Exercise (1) is Riemann integrable, without stating that Theorem 7.2.9 gives you this result. That is, prove this as if Theorem 7.2.9 didn't exist.