MATH 352: HOMEWORK 7
DUE TUESDAY MARCH 22

Re-read Chapter 6

(1) Give an example of a set of polynomial functions, $\{f_n\}$, on $[0,1]$ so that
$$\sum_{n=1}^{\infty} f_n(x) = f(x) \text{ pointwise on } [0,1]$$
but f is not continuous.

(2) Give an example of a set of differentiable functions, $\{f_n\}$, on $[0,1]$ so that
$$\sum_{n=1}^{\infty} f_n'(x) = g(x) \text{ uniformly on } [0,1]$$
but
$$\sum_{n=1}^{\infty} f_n(x)$$
does not converge for all $x \in [0,1]$.

(3) Give an example of a set of differentiable functions, $\{f_n\}$, on $[0,1]$ so that
$$\sum_{n=1}^{\infty} f_n'(x) = g(x) \text{ pointwise on } [0,1]$$
but even though for all $x \in [0,1]$ so that
$$\sum_{n=1}^{\infty} f_n(x)$$
converges to $f(x)$, $f'(x) \neq g(x)$.

(4) Determine the radius of convergence for the following power series and the domain of the limit function. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n} x^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{3} (x-2)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(5) Tell me your exact current grade percentage (to 4 decimal places) in this class.

(6) In this example, we want to explore $C^{\infty}(\mathbb{R})$

(a) Write the set definition of $C^{\infty}(\mathbb{R})$

(b) Show that $C^{\infty}(\mathbb{R})$ is a vector space (Hint: It turns out you need only show subspace properties).

(c) Find a basis $\mathcal{B} = \{f_n\}$ for $C^{\infty}(\mathbb{R})$. Show that every function can be written as a series $\displaystyle\sum_{n=1}^{\infty} a_n f_n$.