MATH 352: HOMEWORK 6
DUE TUESDAY MARCH 8 (<u>DATE CHANGED DUE TO EXAM</u>)

Finish reading Chapter 6

(1) Do Exercise 6.3.4 from your book.
(2) Prove Corollary 6.4.5 from your book.
(3) Let $X = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$ and define $d(f,g) = \sup\{|f(x) - g(x)| : x \in [0,1]\}$.
  (a) Show that (X, d) is a metric space.
  (b) Show that any finite subset of X is equicontinuous.
  (c) Show that if (f_n) is a convergent sequence in X, then (f_n) is equicontinuous.
  (d) We know that a closed and bounded subset of $\mathbb{R}$ is compact. This is not necessarily the case outside of $\mathbb{R}$. Show that a subset of X that is closed, bounded, and equicontinuous is compact.
(4) (**Added Problem based on class discussion**) Prove: $\forall n \in \mathbb{N}$

$$x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots + x^2y^{n-3} + xy^{n-2} + y^{n-1}).$$

(5) (**Problem added with new due date**) Let a_n be a decreasing (not necessarily strictly decreasing) positive sequence. Let $\sum_{n=1}^{\infty} b_n$ be a series so that $\exists M > 0$ so that $\sum_{n=1}^{k} b_n \leq M$ for all $k \in \mathbb{N}$. Then $\forall n \in \mathbb{N}$, $\sum_{n=1}^{k} a_n b_n \leq 2Ma_1$.