

**MATH 352: HOMEWORK 5**  
**DUE TUESDAY FEBRUARY 23**

Finish reading Chapter 5 and start Chapter 6.

MGC #2 Let  $f_n(x) = \frac{nx}{3+nx^2}$ .

- (a) Find the pointwise limit of  $(f_n)$  for all  $x \in (0, \infty)$ .
  - (b) Is the convergence uniform on  $(0, \infty)$ ? Justify.
  - (c) Is the convergence uniform on  $(0, 1)$ ? Justify.
- (1) Prove or disprove: If  $f_n$  is uniformly continuous on  $A$  for all  $n$  and  $f_n \rightarrow f$  on  $A$ , uniformly, then  $f$  is uniformly continuous on  $A$ .
  - (2) Prove or disprove: If  $f_n \rightarrow f$  uniformly on  $A$  and  $f_n \rightarrow f$  uniformly on  $B$ , then  $f_n \rightarrow f$  uniformly on  $A \cup B$ .
  - (3) Prove or disprove: If  $f : A \rightarrow \mathbb{R}$  is a bounded function and  $g_n \rightarrow g$  uniformly on  $A$ , then  $fg_n \rightarrow fg$  uniformly on  $A$ .
  - (4) Prove Theorem 6.2.5 from your book.
  - (5) Prove Theorem 6.3.2 from your book.