MATH 352: HOMEWORK 4
DUE TUESDAY FEBRUARY 16

Read Sections 5.1 and 5.2.

(1) Show that a function $f : A \to \mathbb{R}$ is differentiable at $c \in A$ if and only if

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}.$$

(That is, show that the definition they make you use in Calculus 1 is the same as the definition given in your text.)

(2) Prove or disprove: If $f : A \to \mathbb{R}$ is differentiable on A, then for every $c \in A$ and for every $\varepsilon > 0$, there exists $\delta > 0$ so that whenever $|x - c| < \delta$,

$$|f(x) - f(c)| \leq |f'(c)||x - c|.$$

(3) Let

$$h_p(x) = \begin{cases} x^p \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

(a) Prove or disprove: For every $p > 1$, h_p is continuous on $\mathbb{R}$.

(b) Find p so that h_p is differentiable on $\mathbb{R}$, but h_p' is unbounded on $[0, 1]$.

(c) Find p so that h_p is differentiable on $\mathbb{R}$ with h_p' continuous, but not differentiable at 0.

Be sure to justify all of your answers with proofs.

(4) Prove: If f is differentiable on an interval (a, b), and if y_0 satisfies $f'(a) < y_0 < f'(b)$, then there exists a point $c \in (a, b)$ where $f'(c) = y_0$.