MATH 352: HOMEWORK 2
DUE TUESDAY FEBRUARY 2

(1) Prove or disprove: Suppose f satisfies the property that for every $a \in \mathbb{R}$ and every $\varepsilon > 0$, there exists $\delta > 0$ such that whenever $|x-a| < \delta$, $|f(x) - f(a)| < \varepsilon$ then f also satisfies the property that for every $\varepsilon > 0$, there exists a $\delta > 0$ so that whenever $x, a \in \mathbb{R}$ and $|x-a| < \delta$, $|f(x) - f(a)| < \epsilon$. What about the converse? Is it true or false? Justify your answer with a proof.

(2) Prove or disprove: Suppose f satisfies the property that there exists an $L > 0$ such that whenever $x, a \in \mathbb{R}$, $|f(x) - f(a)| \leq L|x-a|$. Then f also satisfies the property that for every $a \in \mathbb{R}$ and every $\epsilon > 0$, there exists $\delta > 0$ such that whenever $|x-a| < \delta$, $|f(x) - f(a)| < \varepsilon$. What about the converse of this statement? Is it true or false? Justify your answer with a proof.

(3) Show that $f(x) = x^3$ is continuous, but not uniformly continuous.

(4) Suppose $h : (a,b) \to \mathbb{R}$ is is uniformly continuous on $(a, x_0]$ and also on $[x_0, b)$. Prove or disprove that h is uniformly continuous on (a,b).

(5) Given $f : X \to \mathbb{R}$ and $g : Y \to \mathbb{R}$, assume also that $f(X) \subseteq Y$.
   (a) Define $f(X)$.
   (b) Define $f \circ g$.
   (c) Prove or disprove: If f is continuous at $a \in X$ and g is continuous at $f(a) \in Y$, then $g \circ f$ is continuous at $a \in X$.

(6) Let K be a compact subset of $\mathbb{R}$ and let $f : K \to \mathbb{R}$ be continous.
   (a) Define what it means for f to attain a maximum value in K and what it means for f to attain a minimum in K.
   (b) Prove or disprove: f attains both a maximum and a minimum value in K.
   (c) Is it necessary that K is compact? Can it be only closed instead? Justify your answer with a proof.

MGC#1 Prove or disprove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.