MATH 351 – SEPTEMBER 15, 2015 EXERCISE

Statement 1. *If* $S = \left\{1 - \frac{1}{n} \middle| n \in \mathbb{N}\right\}$ *then* $\sup S = 1$, $\min S = \inf S = 0$.

proof: First we will show that $\sup S = 1$. Let $n \in \mathbb{N}$. Then $n > 0$. Thus, $\frac{1}{n} > 0$. So, we have that $1 - \frac{1}{n} < 1$. So 1 is an upper bound of S. Now, let $\epsilon > 0$. We want to show that $\exists s \in S$ so that $1 - \epsilon < s$. Indeed, by the Archimedean property, we know that $\exists n \in \mathbb{N}$ so that $\frac{1}{n} < \epsilon$. Thus,

$$1 - \epsilon < 1 - \frac{1}{n} \in S.$$

Thus, by Lemma 1.3.7, $\sup S = 1$.
Now, we will show that $\min S = 0$. Notice that $0 = 1 - \frac{1}{1} \in S$. We will show that $1 - \frac{1}{n} < 1 - \frac{1}{n+1}$. That is, $1 - \frac{1}{n}$ increases as n gets larger. We know that $n < n + 1$. Thus, $\frac{1}{n+1} < \frac{1}{n}$. So,

$$1 - \frac{1}{n} < 1 - \frac{1}{n+1}.$$

So, $0 \leq 1 - \frac{1}{n}$ for all $n \in \mathbb{N}$. Thus, $\min S = 0$. Now, since $\min S$ exists, we know that $\inf S = \min S = 0$. $\square$