

**MATH 351 Fall 2015 Homework 8****Due: Tuesday 11/10**

Read Chapter 2 sections 3,4. For each of the following problems, use the metric space  $(\mathbb{R}, d)$  where  $d$  is the usual metric. Below MGC stands for “Must Get Correct.”

- (1) Let  $(a_n), (b_n), (c_n) : \mathbb{N} \rightarrow \mathbb{R}$  be sequences so that for each natural number  $n$ ,  $a_n \leq b_n \leq c_n$ . Suppose also that  $(a_n) \rightarrow a$  and  $(c_n) \rightarrow c$ .
  - (a) Prove or disprove:  $(b_n)$  converges.
  - (b) Prove or disprove: If  $(b_n) \rightarrow b$  then  $a \leq b \leq c$ .
  - (c) Prove or disprove: If  $(b_n) \rightarrow b$  then  $a = b = c$ .
  - (d) Prove or disprove: If  $a = c$  then  $(b_n)$  converges.
- (2) Given the sequence  $(a_n) : \mathbb{N} \rightarrow \mathbb{R}$ .
  - (a) Prove or disprove: If  $(a_n) \rightarrow a \in \mathbb{R}$  then  $(|a_n|) \rightarrow |a|$ .
  - (b) Prove or disprove: If  $(|a_n|) \rightarrow |a| \in \mathbb{R}$  then  $(a_n) \rightarrow a$ .

(MGC #2) Prove or disprove: Let  $(a_n) : \mathbb{N} \rightarrow \mathbb{R}$  be a convergent sequence then  $(a_n)$  is bounded.

(MGC #3) Let  $(a_n) : \mathbb{N} \rightarrow \mathbb{R}$  be the sequence whose terms are given by

$$a_n = \frac{3n - 4}{n + 2}.$$

Show that  $(a_n)$  converges.