MATH 351 Fall 2015 Homework 5 Due: Tuesday 10/06

Reread Sections 1.4 and 1.5 in your book.

(1) Prove or Disprove: Suppose $S \subset \mathbb{R}$ is nonempty and bounded above with no maximum element, and suppose $s \in S$. Then $\sup S = \sup(S \setminus \{s\})$.

(2) Prove and extend: If $A_1, A_2, \ldots, A_m$ are each countable sets, then

$$\cup_{k=1}^{m} A_k \quad \text{is also countable}$$

(3) Prove or Disprove: Suppose that $a, b, c \in \mathbb{R}$. Suppose also for all positive real numbers α, β that $|a - c| < \alpha$ and $|a - b| < \beta$. Show that $b = c$.

(4) Let $A = \{a, b\} \subset \mathbb{R}$. Define the set S to be the set of all sequences whose terms are in A. That is,

$$S = \{(x_1, x_2, x_3, ...)|x_n \in A\}.$$

Show that S is uncountable.

(5) Is the set of functions defined by $\{f : \{0, 1\} \to \mathbb{N}\}$ countable or uncountable? Justify your answer with a proof.