

**MATH 351 Fall 2015 Homework 4****Due: Thursday 10/01**

Read Sections 1.4 and 1.5 in your book.

- (1) Prove or Disprove: If  $X \subset \mathbb{R}$  and the set of upper bounds of  $X$  is  $[3, \infty)$  then  $2 \in X$ .
- (2) Let  $c \in \mathbb{R}$  and, for  $S \subset \mathbb{R}, S \neq \emptyset$  be bounded, define the set

$$c - S = \{c - s | s \in S\}.$$

Show that  $\sup(c - S) = c - \inf(S)$ 

- (3) Given sets  $A, B \subset \mathbb{R}$ , we define the set  $A + B$  to be the set of all sums of elements of  $A$  with elements of  $B$ . Prove and extend or disprove and salvage: If  $A$  and  $B$  are nonempty, at most countably infinite, and bounded above, then  $\sup(A + B) = \sup A + \sup B$ .
- (4) Prove that for all  $x \in \mathbb{R}^+$  there exists a number  $\beta \in \mathbb{R}$  satisfying  $\beta^2 = x$ .