MATH 351 Fall 2015 Homework 3 Due: Tuesday 9/29

Read Sections 1.3,1.4 in your book.

(1) In the last homework, we would have been happier with the following result. Prove it.

Given an interval of the form $I = [a, b] \subset \mathbb{R}$. If $a < b$, then $\max I$ and $\min I$ exist and are not equal.

(2) Prove or disprove:
   (a) For all $a, b \in \mathbb{R}$, $|ab| = |a||b|$.
   (b) For all $a \in \mathbb{R}$, $|-a| = |a|$.

(3) Prove or disprove: For all $a, b \in \mathbb{R}$ and $\epsilon > 0$,

$$|a - b| < \epsilon \quad \text{if and only if} \quad b - \epsilon < a < b + \epsilon.$$

(4) What fails in the proof of Theorem 1.4.1 if $I_n = (a_n, b_n)$?

(5) Prove: For every two real numbers a and b with $a < b < 0$, there exists a rational r satisfying $a < r < b$.

(6) Prove: Given any two real number $a < b$, there exists an irrational number t satisfying $a < t < b$.

(7) Prove that $\exists x \in \mathbb{R}$ so that $x^2 = 3$

(8) Prove that $\inf\left\{ \frac{1}{n} \middle| n \in \mathbb{N} \right\} = 0$.

(9) Prove or disprove: If $X \subset \mathbb{R}$ and the set of lower bounds for X is $(-\infty, 2)$, then $3 \in X$.

(10) Prove or disprove: If $X \neq \emptyset$ is finite, then $\max X$ and $\min X$ exists.

(11) This is a "Must Get Correct" problem. If you don't know what that means, you should talk to me.
Given the set

$$A = \left\{ 1 - \frac{n-1}{n+3} \middle| n \in \mathbb{N} \right\}.$$

Find $\inf A$ and then prove your result.