MATH 351 Fall 2015 Homework 2 Due: Tuesday 9/15

Read Sections 1.1, 1.2, and 1.3 in your book.

(1) Let's talk about fields. In your reading (section 1.1) you were given an definition of a field, but it wasn't highlighted as a definition.
   (a) Write the definition of a field by listing the properties both in words and using mathematical symbols.
   (b) List all of the properties that fail for $\mathbb{N}$, the Natural Numbers.
   (c) List all of the properties that fail for $\mathbb{Z}$, the Integers.

(2) One of the most useful tools for Real Analysis is the triangle inequality. Let's discuss it!
   (a) First state the triangle inequality for two points on the real line. Draw a picture that represents (accurately) the triangle inequality on the real line.
   (b) Now, state the triangle inequality for distances in the cartesian plane. Draw an accurate picture that represents this result as well.
   (c) Prove "reverse triangle inequality" that says
   $$|a| - |b| \leq |a - b|.$$

(3) Let's talk about the boundedness of sets.
   (a) Label each of the following sets as finite, bounded below, bounded above, bounded, or unbounded.
      i. $\mathbb{N}$ ii. $\{x \in \mathbb{Z} | x^2 \leq 5\}$ iii. $\{x \in \mathbb{Q} | x^2 \leq 5\}$ iv. $\{x \in \mathbb{R} | x^2 < 5\}$

      v. $(0, 2)$ vi. $[0, 2]$ vii. $(0, 2) \cap \mathbb{Q}$ viii. $[0, 2] \cap \mathbb{Q}$

      ix. $(0, 2) \cap \mathbb{Z}$ x. $[0, 2] \cap \mathbb{Z}$ xi. $\{\frac{n-1}{n} | n \in \mathbb{N}\}$
   (b) For each of the above sets, state a maximum and a minimum when it exists.
   (c) For each of the above sets, state an upper bound and a lower bound when such a bound exists.
   (d) For each of the above sets, state a least upper bound and a greatest lower bound when such a bound exists.
   (e) Finally, prove or disprove the following statement. If it is false, salvage the statement keeping as much the same as possible and prove your new statement.
   Let $a, b \in \mathbb{Q}$ and $S = \{x \in \mathbb{Q} | a \leq x^2 \leq b\}$, then the greatest lower bound of S and least upper bound of S exist in $\mathbb{Q}$.

(4) Given a set $A = \{\frac{2n-3}{3n+1} | n \in \mathbb{N}\}$
   (a) Find the $\sup A$
   (b) Prove your findings in part a.