MATH 351 Fall 2015 Homework 10 Due: Tuesday 11/24

Read Chapter 2 sections 5,6,7. For each of the following problems, use the metric space $(\mathbb{R}, d)$ where d is the usual metric (unless directed otherwise).

(1) Let $d\left((x_1, y_1), (x_2, y_2)\right) = \max\left\{|x_1 - x_2|, |y_2 - y_1|\right\}$.
  (a) Show that $(\mathbb{R}, d)$ is a metric space.
  (b) Draw a picture of $\mathcal{N}_1(0,0)$.

(2) Give an example (if possible) of a sequence for each of the following. If it is not possible, prove it.
  (a) A sequence that is Cauchy and nonmonotone.
  (b) A sequence that is Cauchy and unbounded.
  (c) A sequence that is Cauchy and has a subsequence that is not Cauchy.
  (d) A sequence that is not Cauchy and has a subsequence that is Cauchy.

(3) Show

$$\text{If} \quad \sum_{n=0}^{\infty} a_n \quad \text{and} \quad \sum_{n=0}^{\infty} b_n \quad \text{both converge}$$

$$\text{then} \quad \sum_{n=0}^{\infty} c_n \quad \text{converges,}$$

where $c_n = a_n + b_n$.

(4) Theorem 2.6.2 cannot be used until we have a proof. The outline is on page 59 in your book. Exercise 2.6.2 asks for the details. Give a proof of theorem 2.6.2 here.

(5) Let $(a_n) : \mathbb{N} \to \mathbb{R}$ be a sequence. (Assume $\mathbb{R}$ is endowed with the standard metric.)
  (a) Prove that if (a_n) does not have an increasing subsequence, then it must have a largest term.
  (b) Prove that if (a_n) does not have a decreasing subsequence, then it must have a smallest term.

(6) Show that every sequence in $\mathbb{R}$ has a monotone subsequence.

(7) Using the previous question, what can you say about bounded sequences? Prove your statement.