BIG OPPORTUNITY   SPRING '06

The big O. One last opportunity!

No calculators or cell phones are allowed. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. The idea underlying all of Calculus, including every single topic we covered this year, is that of the limit.

In a paragraph or two, explain what a limit is. Give at least three examples of topics in Calculus that used limits, and how the concept of limit is used in each case.

2. Find the following limits:

a) $\displaystyle\lim_{n\to\infty} \frac{\sin n}{n}$

b) $\displaystyle\lim_{h\to 0} \frac{\sin h}{h}$

c) $\displaystyle\lim_{n\to\infty} \frac{n^2}{2n^2 - \pi n + 13}$

d) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2^k}{3^{k+1}}$

3. Look at the (possibly infinite) region surrounded by the negative y-axis, the positive x-axis, and the function $y = \ln x$.

Write an integral which would calculate the area of this region. Be sure to use limit notation correctly. (You **do not** have to calculate this integral.)

Write an integral which would calculate the volume generated by spinning this region around the y-axis. Again, be sure to use limit notation correctly.

This time, you have to do the integral. Is this volume finite or infinite? Explain.

4. For what values of x does the following power series converge? Be complete - and at all points where the series converges, determine if it converges absolutely or conditionally.

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n+1}}$$

5. Find the Maclaurin Series for the following functions:

$f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$

$g(x) = \frac{x}{(1+x)^2}$

6. Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely.

How many terms of this series would you need to take to be within $\frac{1}{100}$ of the eventual limit?

Tell me the best and worst things about this year at St. Mary's.

7. Integrate:

a) $\displaystyle\int y\, e^{-2y}\, dy$

b) $\displaystyle\int \frac{1}{x^2 + x}\, dx$

Extra Credit: This past week a painting sold for $95.2 million, the second highest amount ever paid for a painting. Name the artist and title of this painting.