

OPPORTUNITY 1 – $2e^{i\pi}$

SPRING '06

Yipee! Another Opportunity!

No calculators or cell phones are allowed. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Find the limit of each sequence below. Show your work.

$$a_n = \frac{n^{497}}{n^{497} + 3n^2 - \pi}$$

$$b_n = \frac{\ln n}{n + \sqrt{n}}$$

$$c_n = 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^n}$$

$$d_n = \frac{\cos n}{(n+1)(n+2)}$$

2. What is a sequence? What does it mean for a sequence to converge?

What is a series?

Explain how the convergence of sequences and series are related.

3. Find the sum of the following series. $\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$

4. True or False? For each of the following, determine if the statement is True or False. If True, explain why. If False, give an example showing why it's false.

a) If a sequence a_n is bounded, then it converges.

b) If a series $\sum_{n=1}^{\infty} b_n$ converges, then the sequence b_n converges.

c) If a sequence c_n converges to a real number $c > 0$, then the series $\sum_{n=1}^{\infty} c_n$ converges.

d) If a function $f(x)$ is positive and $\sum_{n=1}^{\infty} f(n)$ converges, then $\int_1^{\infty} f(x) dx$ also converges.

5. Does each series below converge or diverge? Prove your answer, noting any tests you use.

a. $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

b. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5 + 3n - 2}}$

c. $\sum_{n=1000}^{\infty} \frac{1}{n}$

d. $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

e. $\sum_{n=1}^{\infty} \frac{n + \cos n}{e^n}$

f. If you could spend one year somewhere,

where would you choose and why?

6. Suppose $f(x)$ is a positive, decreasing function. Put the following in order from least to greatest:

$$\int_{583}^{\infty} f(x) dx \quad \sum_{n=583}^{\infty} f(n) \quad \int_{582}^{\infty} f(x) dx$$

Draw a picture to illustrate your reasoning and explain your thinking in a paragraph (referring to your picture).

Extra Credit: Paul Sarbanes has represented the state of Maryland in the US Senate since 1977 but will retire next January. Name any of the major party candidates running for his seat. (1 point each, up to a maximum of 4).