

Yay! You get to show off your knowledge!

No calculators or cell phones are allowed. If you have any questions, please ask Dave. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. How do we define the natural log function,  $f(x) = \ln x$ ? (Hint: think areas.)

Why is  $(\ln x)' = \frac{1}{x}$ ?

2. Find the derivative of  $f(x) = \arccos x$  and prove it. Explain your steps.

What is the derivative of  $g(x) = \arccos(e^{2x})$ ?

3. We'll start with some derivatives. In each of the following, find  $y'$  (a.k.a.  $\frac{dy}{dx}$ ).

a)  $y = \ln(\sin x)$

b)  $y = e^{\sqrt{x}}$

c)  $\ln(xy) = \cos x$

d)  $y = \ln(x^2 + 3x + \pi)$

e)  $y = \int_1^{\cos x} e^{t^2} dt$   
far.

f) Tell me about the best day of your life so far.

4. Integrate the following:

a)  $\int \frac{1}{1+2x+x^2} dx$

b)  $\int \frac{1}{2+x^2} dx$

c)  $\int \frac{2x+3}{x^2+3x-5} dx$

d)  $\int \frac{2x+5}{x^2-x-2} dx$

e)  $\int \frac{1}{1-x^2} dx$

f)  $\int \sin^3 x \cos^5 x dx$

g)  $\int \frac{\ln(1+x)}{1+x} dx$

h)  $\int xe^x dx$

5. For each of the following determine if the integral converges or diverges. If possible, find the value of the integral.

a)  $\int_1^{\infty} \frac{1}{1+x^3} dx$

b)  $\int_1^{\infty} \frac{1}{x\sqrt{x^2-1}} dx$

c)  $\int_0^{\pi/2} \tan(x) dx$

6. a) Define in your own words what it means for a sequence  $a_n$  to converge to a limit  $L$ .

Referring back to your definition, explain the convergence or divergence of each of the following.

b) 1, 1.1, 1, 2.01, 1, 3.001, 1, 4.001, 1, ...

c)  $b_n = \frac{1}{n^2+1}$

d)  $d_n = (-\frac{1}{2})^n$

*Extra Credit:* As noted in class today, "Crash" won the Oscar for Best Picture last night. Name the other four films which were nominated in this category.