Calculus II, Spring '05 Name:_____

Final Opportunity

No calculators or cell phones are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Suppose we take the function $f(x) = \frac{1}{x^{2/3}}$ between $x = 1$ and $x = \infty$, and rotate it around the x-axis. We could calculate the volume of this using either the Shell method or the Washer/Disk method. Write both, being sure to use limit notation for the improper integrals. Calculate one or the other to determine the volume of the solid.

Shell Method

Washer/Disk Method

2. Do the following integrals.

a) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

b) $\int \frac{1}{\sqrt{1-x^2}}\,dx$

c) $\int \frac{1}{1-x^2}\,dx$

3. Determine whether each series below converges conditionally, converges absolutely, or diverges.

a) $\sum_{n=1}^{\infty} \frac{2\pi^{n-1}}{4^n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+1}}$

c) $\sum_{n=1}^{\infty} \frac{1+\cos(2n)}{1+n^{1.1}}$

4. Prove the following limit, justifying your reasoning at ever step.

$$\lim_{n\to\infty} (1+\frac{1}{n})^n = e.$$

5. Find the Taylor Series near $x = 0$ for the function $f(x) = \frac{x}{1-x^3}$. For full credit, write your answer using summation notation.

6. Consider the differential equation $y'(x) = \frac{1}{y}$.

Draw the direction field for this equation on the given axes.

Solve the equation using the Separation of Variables technique.

Find the solution which goes through the point $(-1, 1)$, and graph it on your direction field above.

7. Taylor Series. Derive the Taylor series for the function $f(x) = \sin x$ near the point $x = \pi/2$. Graph $T_0(x)$ and $T_2(x)$ on the axes shown. Show your work.

8. And finally...

Give an example of a use of the Fundamental Theorem of Calculus, part I.

Give an example of a use of the Fundamental Theorem of Calculus, part II.

Extra Credit: Yesterday, elections were held in the United Kingdom. Tony Blair's party managed to hold on to power, despite losing seats in Parliament because of Britain's support of the US in the war in Iraq. Name Tony Blair's party, along with the two other largest political parties in the U.K.