

Opportunity III: Revenge of the Series

Mmmmmm...Lasagna.

No calculators or cell phones are allowed. If you have any questions, please ask me or Karina. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. Answer some general questions about sequences and series.

- What's a sequence? Give an example.
- What's a series? Give an example.
- Does the sequence $a_n = \frac{n-1}{n+1}$ converge or diverge? Explain.
- Does the sequence $s_n = \sum_{k=1}^n \frac{1}{k}$ converge or diverge? Explain.

2. Explain the Limit Comparison Test and why it works. Give an example to illustrate your explanation.

3. Find the sum of the two series below:

- $\sum_{n=2}^{\infty} \frac{2\pi^n}{4^{n-1}}$
- $\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$

4. Here is the picture which illustrates the Integral Test. $f(x)$ is a positive, decreasing function.

The following are four different quantities to be compared:

$$A = \sum_{n=1}^{\infty} f(n) \quad B = \int_1^{\infty} f(x) dx \quad C = \sum_{n=2}^{100} f(n) \quad D = \sum_{n=2}^{\infty} f(n).$$

For each pair, make a comparison, writing \leq , \geq , or ? (for Can't Tell). Explain your reasoning.

A B

A C

A D

B C

B D

C D

5. Completely determine the convergence and divergence of the power series below. That is, determine for which values of x the series converges absolutely, converges conditionally, and diverges. Show your work.

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+2)4^n}$$

6. Determine whether the following series converge or diverge. Give reasons for each.

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

b) $\sum_{n=1}^{\infty} \frac{\arctan n}{n\sqrt{n}}$

c) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

d) $\sum_{n=1}^{\infty} \frac{1}{n2^n}$

e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}-1}$

f) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

Extra Credit: According to President Bush, the Social Security system will “go bankrupt” in 2017. Explain what exactly will happen in that year, according to government projections.