

Opportunity II

Make me cook you dinner. Please. I promise I'll cook well.

No calculators or cell phones are allowed. If you have any questions, please ask me or Karina. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

1. What is the number e ?

Find derivatives of the following:

$$y = e^{\cos x}$$

$$y = \ln(e^{\sqrt{x}})$$

2. Show that the derivative of $\arctan x$ is $\frac{1}{1+x^2}$. Give a reason for each step.

Let $f(x) = \arctan(\sin x)$. Find $f'(x)$.

3. Integrate the following:

$$\int e^x dx$$

$$\int xe^x dx$$

$$\int \sin^3 x \cos^3 x dx$$

$$\int xe^{x^2} dx$$

$$\int \frac{e^x}{1+e^x} dx$$

$$\int \frac{1}{x\sqrt{(x^2-1)}} dx$$

4. Integrate the following:

$$\int_1^\infty \frac{x}{(x^2+1)^{3/2}} dx$$

$$\int \ln(1+x^2) dx$$

$$\int \frac{2x^2+x}{(x-2)(x^2+1)} dx$$

$$\int x^2 \sin(x) dx$$

What do you want to be when you grow up?

5. For each sequence below, give its limit or state why the limit doesn't exist. Explain your reasoning.

$$a_n = \frac{\pi n}{2n+1}$$

$$b_n = \sin\left(\frac{\pi n}{2n+1}\right)$$

$$c_n = \frac{n^2-1}{en^3+\sqrt{n}+\pi^n}$$

$$d_n = \int_1^n \frac{1}{t^2} dt$$

6. Here is the function $f(x) = \frac{1}{x\sqrt{x-1}}$.

Does the integral $\int_1^\infty f(x) dx$ converge or diverge? Explain.

Extra Credit: Yesterday was the 40th anniversary of "Bloody Sunday." What happened on that day?