Calculus II, Spring '04 Name:____________________

Opportunity III

Here you go! It's your last chance before the final to show me (OK - show Kristine) what you've learned this semester. As before, you have as much time as you need, but you may not use a calculator, cell phone, or spare brain. In all cases, explain your work - if you use a test or theorem, say so.

1. (20 points - 5 pts each)

Do the following sequences converge or diverge? If they converge, say what the limit is. Give reasons for your answers.

a) $1, 0, \frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}, 0, \ldots$

b) $s_n = \displaystyle\sum_{k=1}^{n} \frac{1}{k}$

c) $a_n = \frac{(-1)^n}{\sqrt{n}}$

d) $b_n = (1 - \frac{1}{n})^n$

2. (10 points)

Here's the picture that goes with the Integral Test:

State the Integral Test, including hypotheses.

Use the picture and the ideas behind the Integral Test to prove:

$$\frac{\pi}{4} \leq \sum_{n=1}^{\infty} \frac{1}{1+n^2} \leq \frac{\pi}{4} + 1.$$

(Hint: $\displaystyle\lim_{N\to\infty} \arctan N = \frac{\pi}{2}$ and $\arctan(1) = \frac{\pi}{4}$.)

3. (10 points) Find the sum of the two series below. Explain your answers.

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$$

$$\sum_{n=1}^{\infty} \frac{3\pi^n}{2^{2n+1}}$$

4. (10 points)

We all know now that a sequence is a list of numbers and a series is when you add them up.

What is the definition of a sequence converging to a limit? (i.e. what does $\displaystyle\lim_{n\to\infty} a_n = L$ mean?) Explain.

What is the definition of a series converging to a limit? (i.e. what does $\displaystyle\sum_{k=1}^{\infty} b_k = M$ mean?) Explain.

5. (30 points - 5 pts each)

Do the following series converge or diverge? If a series converges, say whether it converges absolutely or conditionally. Give reasons for your answers.

$a) \displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}\sqrt{3n-1}}$

$b) \displaystyle\sum_{n=1}^{\infty} \frac{n(-3)^n}{4^{n-1}}$

$c) \displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ (note: You cannot cancel the n's.)

$d) \displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}$

$e) \displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(2n-1)}$

$f) \displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

6. (10 points)

Completely determine the behavior of the power series below. That is, show for which x it converges absolutely, diverges, and converges conditionally.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{\sqrt{n}\, 3^n}.$$

7. (10 points)

Find the power series for the following two functions, justifying your answers:

$\dfrac{2x}{1+x^2}$

$\ln(1+x^2)$

Extra Credit: A new branch of the Smithsonian museum is set to open soon, filling one of the last spots along the Mall. It stands directly across the Mall from the East Wing of the National Gallery of Art. For one point, name the new museum. For a second point, name the architect who designed the East Wing.