Calculus II, Spring '04 Name:_____

Opportunity II

Here you go! It's another chance to show me what you've learned so far this semester. No calculators or cell phones are allowed. If you have any questions, please ask me. Explaining your reasoning will help you earn partial credit if your answer isn't entirely correct. Please write clearly and legibly; scratch paper will be available.

BTW, $\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + c$.

1. Suppose the region below the curve $y = e^{-x}$ and above the positive x-axis is rotated around the x-axis. Is the resulting volume infinite or finite? Explain.

2. Compute the following integrals. Note that for definite integrals, your answer should be a number.

$\int \frac{\cos x}{\sin^2 x}\, dx$

$\int_0^1 \arctan x \, dx$

$\int \frac{dx}{\sqrt{x^2-4x}}$

$\int \sin^2 x \cos^3 x \, dx$

$\int \ln x \, dx$

$\int_1^2 \frac{\sqrt{1-x^2}}{x}\, dx$

3. Look, a whole page to do one integral! (Please do any messy algebra on scratch paper first, then more clearly on this sheet.)

$$\int \frac{2x^3 - x^2 + 2}{(x^2 - 2x + 2)(x^2 - x)}\, dx$$

4. Find the following limits. Give reasons for your answers. (No reason, no points.)

a) $\lim_{n\to\infty} \frac{3n(n+3)}{(n+1)(n+3)}$

b) $\lim_{n\to\infty} \frac{\ln n}{n^{1/3}}$

c) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

d) $\lim_{n\to\infty} \left(1 + \frac{\sqrt{\pi}}{n}\right)^n$

e) $a_n = \frac{\cos(n\pi)}{(-1)^n}$

f) $b_n = \frac{\cos(n)}{\ln n}$

5. What does it mean when we say that a sequence $\{a_n\}$ converges to 3? Explain in about a paragraph.

6. For each of the following, come up with an example or say why no such example exists.

a) A sequence which is bounded above by 4, bounded below by 1, and divergent.

b) A sequence $\{a_n\}$ which converges, but $\{\frac{1}{a_n}\}$ diverges.

c) A sequence $\{a_n\}$ which converges, but $\{\frac{1}{(a_n)^2+1}\}$ diverges.

7. Do the following integrals converge or diverge? If they converge, evaluate them. If they diverge, explain why.

$\int_0^3 \frac{1}{x^{1/3}}\, dx$

$\int_0^\infty \frac{1}{1+x^2}\, dx$

$\int_1^\infty \frac{1}{(x-1)^4}\, dx$

Extra Credit: In 1991 the U.S. formed a coalition of nations to reverse the Iraqi invasion of Kuwait. Fortunately for the U.S., other nations have not formed coalitions to reverse its invasions. Name the countries invaded by the U.S. in the last 25 years.